Equity participations, hold-up, and firm boundaries
PRELIMINARY

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Abstract

This paper shows how partial equity participations can solve the hold-up problem: by weakening the bargaining position, they make the hold-up less effective, and thus improve investment and specialization incentives of the other party. With one-sided dependency, a 50% participation gives full efficiency. In the case of bilateral dependency, the unique efficient solution is equivalent to a merger. This basis for determining optimal firm boundaries is essentially one of incentive design, as suggested by Holmström (1999), rather than property rights. The theory also shows how joint ventures can sometimes realize the benefits of equity participations, while avoiding some concurrent problems.

1 Introduction

Companies often own significant stakes in other companies. Intel, for example, reported to have more than \$ 8 billion in 'strategic investments' at the end of '99. The key result of this paper is that such equity participations can eliminate the hold-up problem[1]. In particular, with unilateral dependence, the firm that can commit hold-up should take a 50% participation in

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[1]As in the models of Grossman-Hart-Moore (1986, 1990, 1995), we consider a firm that makes incontractible specific investments. Since its partner, through renegotiation, captures some of the quasi-rents, the firm will underinvest. This type of problem is commonly referred to a the 'hold-up problem'. This paper considers specifically the case that the investment costs are borne by the firm (such as investments in plant and equipment or company-paid training programs), which is arguably the most important case in economic terms.

the one that makes the investment. The intuition behind this result is that such equity partipations make the firm internalize its bargaining threats. This weakens the effectiveness of the hold-up and thus increases the other's incentives to invest.

The role of equity participations has been studied mainly in the contexts of technology transfers to developing countries (e.g. Marjit and Mukherjee (1998)) and of supporting collusion (Rodriguez (1991)). With respect to the issues considered in this paper, the most important work is that of Aghion and Tirole (1994) who study the allocation of ownership in R&D ventures. In their model, a researcher spends private effort and is compensated by the licencing fee he can obtain. For that case, they argue that equity participations are irrelevant to the underinvestment problem. Our paper comes to the opposite conclusion for the case that the investment costs are borne by the firm. Note that this includes the case of private but contractible effort. On the empirical side, Pisano (1989) concluded in a study of the biotech industry that equity participations are more likely in cooperations that involve specific investments. This result is in line with our results. Since the direction of dependencies and participations was not specified, however, it did not test the finer predictions of this paper.

In a related strand of literature, Noldeke and Schmidt (1998) show how options on the equity of the other firm can solve the hold-up problem when the investments are sequential. Dasgupta and Tao (1998) present a very particular model in which equity participations that are sold after making the specific investment but before renegotiation, partially alleviate the hold-up problem. Hellmann (1997) shows that the possibility of incontractible actions can be an important determinant for the source of equity financing of start-ups.

After discussing in section 2 the impact that equity participations have on bargaining and after deriving in sections 2-5 how they solve the hold-up problem, the paper shows in section 6 that this solution also improves the incentives to specialize and can align the interests of different parties. Relying on the analysis of Freeland (2000), it further argues that the early GM-Fisher Body relations are consistent with the theory developed here. Sections 7 and 8 study extensions to more than one dependence relationship. The most interesting result here is that the unique optimal solution in the case of bilateral dependency turns out to be equivalent to a merger. The paper therefore suggests a perspective on firm boundaries that is based on the incomplete contracting approach but differs fundamentally from the property rights perspective, suggested by GHM. The driving force in this case is the ability of a firm to set its managers' incentive schemes. *(What* *follows has to be confirmed with a formal model.)* In particular, in the situation considered, two separate firms are faced with a prisoner's dilemma in setting the incentives, with each firm having a private incentive to secretly side-contract with its manager. The motive behind the merger is thus essentially incentive design, in line with Holmström (1999). Finally, section 9 shows how joint ventures often provide a solution when the suggested participations are problematic or impossible. All proofs are in appendix.

2 Managerial objectives and altruistic bargaining

Throughout, this paper maintains the assumption that:

Assumption 1 *Management pursues 'shareholder value maximization', i.e. it maximizes the expected value of the discounted cash flows from the firm to its shareholders.*

This will be satisfied, for instance, if managers do not derive any direct personal costs or benefits from the decisions they make, are risk-neutral, and have wages that are a strictly increasing function of the equity value of the firm, e.g. by holding one share in the firm. While this assumption might seem trivial at first, it has a not so obvious but very critical implication: Management does *not* take into account who its shareholders are and what they privately would prefer management to do. In particular, management only cares about the company's own profits, even if doing so might hurt a firm that partly owns the firm. Legally, this is indeed management's fiduciary duty (towards the other shareholders) as long as the other firm owns anything less than 100 % of the shares.

As such, the analysis focuses exclusively on the income aspects of equity participations and makes abstraction from the control aspects. While the latter are obviously very important, this approach is useful for a number of reasons :

1. It allows a very transparent analysis of the implications of 'equity participation as residual income rights'.

2. When taking into account control, it becomes unclear what the objective of management should be : maximizing the utility of the controlling shareholders, that of all shareholders, or something else?

3. As mentioned earlier, the assumption is (explicitly) consistent with the legal doctrine regarding the fiduciary duty of management, and with the fiduciary duty of a controlling shareholder.

4. The assumption perfectly fits non-voting equity participation, as in the GM-Fisher Body case discussed later.

Section 6 discusses some implications of the theory for the control aspects of equity participations.

While assumption 1 implies that management 'does not care who owns its shares', it *does* require management to take into account how its own actions influences the profit of firms in which it has *itself* an equity participation. Since the precise form of the implied managerial objective function is central to the results of this paper, it is worthwhile deriving it in some detail. To that purpose, consider two firms, A and B. Let $\tilde{\pi}_i$ denote firm i's operational profit, prior to any income streams from equity participations the firm might hold, let $\hat{\pi}_i$ denote the firm's overall profit, including such streams, and let η_i denote the equity participation of firm i in the other firm. We have of course $\hat{\pi}_i = \tilde{\pi}_i + \eta_i \hat{\pi}_j$. The most transparent way to derive the correct managerial objective is to focus on the value of the equity held by independent shareholders[2]. Since these investors end up with $(1 - \eta_B)$ of firm A's profits, that value obviously equals $(1 - \eta_B)\hat{\pi}_A$. Combining $\hat{\pi}_A = \tilde{\pi}_A + \eta_A \hat{\pi}_B$ with $\hat{\pi}_B = \tilde{\pi}_B + \eta_B \hat{\pi}_A$ gives

$$\hat{\pi}_A = \frac{\tilde{\pi}_A + \eta_A \tilde{\pi}_B}{1 - \eta_A \eta_B}$$

[2]Of course, it does not matter whether we look at the value of all the equity or only at the value of the equity held by independent investors. But when looking at the value of equity held by the other company, it is easy to start confusing things. An alternative derivation of the result in the text is to 'follow the cash flows'. In particular, consider one dollar of operational profit of firm A. The independent shareholders get a direct share $1 - \eta_B$ of that dollar, while η_B flows to firm B. As a shareholder in B, company A gets a share η_A of that money back, which amounts to $\eta_A \eta_B$ dollars. Independent shareholders get again a direct share $1 - \eta_B$ of that extra income, etc. Continuing this argument, the overall payoff of the independent shareholders of one dollar operational profit going to firm A is

$$(1 - \eta_B)\left(1 + \eta_A \eta_B + (\eta_A \eta_B)^2 + ...\right) = \frac{(1 - \eta_B)}{1 - \eta_A \eta_B}$$

An analogous argument shows that their payoff of one dollar operational profit to firm B is $\frac{\eta_A(1-\eta_A)}{1-\eta_A\eta_B}$. So their overall payoff becomes indeed

$$\frac{(1 - \eta_B)}{1 - \eta_A \eta_B}\tilde{\pi}_A + \frac{\eta_A(1 - \eta_A)}{1 - \eta_A \eta_B}\tilde{\pi}_B$$

The value of the independent shareholders' equity holdings in A, and therefore the objective of A's managers, is thus

$$(1-\eta_B)\frac{\tilde{\pi}_A+\eta_A\tilde{\pi}_B}{1-\eta_A\eta_B}$$

Note that, as required, the sum of these values over both companies equals the total operational cash flow :

$$\begin{aligned}
&(1-\eta_B)\frac{\tilde{\pi}_A+\eta_A\tilde{\pi}_B}{1-\eta_A\eta_B}+(1-\eta_A)\frac{\tilde{\pi}_B+\eta_B\tilde{\pi}_A}{1-\eta_A\eta_B}\\
&= \frac{\tilde{\pi}_A-\eta_A\eta_B\tilde{\pi}_B+\tilde{\pi}_B-\eta_A\eta_B\tilde{\pi}_A}{1-\eta_A\eta_B}\\
&= \tilde{\pi}_A+\tilde{\pi}_B
\end{aligned}$$

Since all the results in this paper are invariant to affine transformations of the managerial objective functions, we can simplify the respective managerial objectives (for given η_i) to[3] :

$$\begin{aligned}
u_A &= \tilde{\pi}_A+\eta_A\tilde{\pi}_B\\
u_B &= \tilde{\pi}_B+\eta_B\tilde{\pi}_A
\end{aligned}$$

The rest of this section considers how such 'utility functions' influence cooperative and non-cooperative bargaining.

Altruistic bargaining Let companies A and B now bargain on whether and at what price to cooperate. Let p be the agreed-upon transfer from company A to B, conditional on cooperation. Let company i's operational profit, *excluding* the transfer p, be $\pi_i(x) \geq 0$, with $x \in \{0,\ 1\}$ indicating whether the firms cooperate and $\pi_i(1) > \pi_i(0)$. Except when η_A and η_B happen to be equal, this bargaining game is not a TU-game, since the sum of utilities depends on the transfer p. It follows that Nash bargaining does not necessarily result in splitting the gains from agreement. Rather, the Nash bargaining solution solves :

$$\begin{aligned}
&\max_p[((\pi_A(1)-p)+\eta_A(\pi_B(1)+p))-(\pi_A(0)+\eta_A\pi_B(0))]\\
&\qquad[((\pi_B(1)+p)+\eta_B(\pi_A(1)-p))-(\pi_B(0)+\eta_B\pi_A(0))] \qquad (1)
\end{aligned}$$

[3]These objective functions might seem to allow managers to create value out of nothing with some creating accounting. Consider for example the case that firm A pays a dollar to firm B. Since A owns part of B, it pays in effect only part of the dollar. But B gets the full dollar. Combining this with a symmetric argument on transfers from B to A it seems possible to create money out of nothing. Obviously, such reasonings must contain some flaw. In this example the issue is of course that, by owning part of A, B also pays part of the dollar it gets.

Notice that a player internalizes (via his equity ownership) part of the loss he causes to the other player by not cooperating. This will be the key force in the later results : owning part of the equity of the other company softens a player's bargaining stance so that he becomes less effective at holding up; realizing that, the other player will be willing to invest more.

To motivate the use of the Nash solution, note that a modification of the results of Binmore-Rubinstein-Wolinsky (1986, henceforth BRW) applies, as we will now show.

Consider first an infinite-horizon alternating-offer bargaining game with a probability of breakdown q after each rejection of an offer. Players make price offers $p \in P$ with P 'large enough' but compact[4]. Company A's payoff is

$$\begin{array}{rl} \pi_A(1) - p + \eta_A(\pi_B(1) + p) & \text{if the parties agree on price } p \\ \pi_A(0) + \eta_A \pi_B(0) & \text{upon breakdown} \end{array}$$

and analogous for B. We then have that

Proposition 1 *As the probability of breakdown q converges to zero, the unique subgame perfect outcome of the game converges to the Nash solution as defined in (1).*

The proofs of this and all other propositions are in appendix.

An analogous result applies to the corresponding δ-discounted bargaining game. Let both players now get their outside option in each period as long as no agreement is reached, and the agreed payoff forever after an agreement. In particular, A gets $(1-\delta)(\pi_A(0) + \eta_A \pi_B(0))$ in each period where the contract is rejected, and analogous for B. Once a price $\hat{p} = (1-\delta)p$ has been agreed, A gets $(1-\delta)((\pi_A(1) - p) + \eta_A(\pi_B(1) + p))$ per period, and again analogous for B. Then :

Proposition 2 *As the agents impatience $(1-\delta)$ decreases to zero, the unique subgame perfect outcome of this game converges to the Nash solution as defined in (1).*

[4] In particular, the proof uses $[\underline{p}, \overline{p}] \subset P$ with $\underline{p}$ and $\overline{p}$ defined by

$$\begin{aligned} \pi_B(1) + \eta_B \pi_A(1) + (1-\eta_B)\underline{p} &= \pi_B(0) + \eta_B \pi_A(0) \\ \pi_A(1) + \eta_A \pi_B(1) - (1-\eta_A)\overline{p} &= \pi_A(0) + \eta_A \pi_B(0) \end{aligned}$$

which implies $\underline{p} \leq 0 \leq \overline{p}$.

3 The basic model

Consider now a firm A that can take on a project, say developing a new product. The project is its sole activity and requires it to invest up-front an amount of resources $I \in \mathcal{R}^+$, at cost $c(I) \geq 0$. We assume :

Assumption 2 *The costs of investments are borne by the firm undertaking it.*

To motivate this assumption, note that most real-life examples in the literature concern investments in physical capital, of which the costs are always borne by the firm (e.g. Hart 1990, p26). Moreover, even the costs of building human and social capital are often directly or indirectly[5] borne by the firm. This category of investments thus arguably constitutes the larger part of the hold-up problem.

In marketing the product, A can work on its own ($x = 0$) or cooperate with company B ($x = 1$). Assume that A and B cannot write a complete ex-ante contract, but negotiate instead a price for B's support at the time the product is launched, according to the Nash bargaining solution. This incontractibility might come from the fact that A is not willing to discuss its plans with B before it has developed and patented the product. It is also difficult to specify ex ante what good marketing of an as yet non-existing product is. Let the companies' discount factor be zero, and B's costs and outside revenues be normalized to zero. The latter is without loss of generality as long as B does not have to make any ex ante investments.

The gross profit, which accrues to A, is $R(I, x)$. We assume the following functional properties :

Assumption 3 *All functions are smooth. $R(I, x)$ is strictly increasing and supermodular, and strictly concave in its first argument. $c(I)$ is strictly increasing and convex. Finally, $R(\hat{I}, 0) - c(\hat{I}) > 0$ $\forall I \leq I^*$ where $I^* =$ $\text{argmax}_I\, R(I, 1) - c(I)$.*

The supermodularity indicates that there are complementarities between A's development effort and B's marketing resources : an extra dollar investment by A pays off more if it gets B's support in marketing the product. The very last part of the assumption, together with the convexity/concavity assumptions and the overall setup of the problem guarantee that there is no

[5]Respectively as, on the one hand, time lost by 'on the job training', formal training programs, visiting suppliers during work hours, and, on the other, as part of 'what you are being paid for'.

risk of bankruptcy for either company, even when negotiations break down. In that case the company can borrow freely at the risk-free interest rate[6], which we assume to equal zero.

The game unfolds as follows :

I. The original shareholders of A bring in equity capital E_A. Firm B can then take an equity participation, in a way discussed in more detail below, and possibly brings in extra capital E_B in return. A's management finally incurs a further debt D with lenders (or bondholders).

II. A's management invests resources I with $c(I) \leq E_A + E_B + D$.

III. A and B negotiate a price p.

IV. A gets $R(I, x)$, pays p to B if $x = 1$ and repays debt.

As long as B's outside option is not affected by I, the results extend to the cases where the benefit $R(I, 1)$ accrues (partially or completely) to B. This extends the model to a wide range of cases : B being just a support in A's marketing, B really distributing the product, A doing contract-research, or A and B jointly distributing the product (e.g. via licencing or OEM agreements). The results also readily extend to the situation where B is a union or a group of critical employees rather than a different company. As such the model also considers the impact of (substantial) ESOP's.

Sometimes we also use the following assumption to guarantee that the problem has an interior maximum :

Assumption 4

$$\begin{aligned} \lim_{I \to \infty} \frac{\partial R(I,1)}{\partial I} - c'(I) &< 0 \\ \frac{\partial R(0,0)}{\partial I} - c'(0) &> 0 \end{aligned}$$

4 The wrong solution

Following the standard GHM argument, it follows that, absent any equity participation, A's ex-ante payoff is

$$\frac{R(I,1) + R(I,0)}{2} - c(I)$$

[6] This implicity assumes that there is a competitive and perfect capital market and that all information is freely available.

so that, by monotone comparative statics and the supermodularity of $R(I, x)$, the firm underinvests relative to the socially efficient level.

One interpretation of this result is that there is a positive externality : B captures part of the benefits of A's investment I. This leads, as always, to underinvestment. It therefore seems that the problem can be alleviated by having A take an equity participation in B. In that case, A should capture a larger share of the returns to its investment, which would improve efficiency. This argument is incomplete, however, in that it does not take into account the fact that the equity participation also changes A's objective function when bargaining. In particular, the participation weakens A's bargaining power and thus reduces the share it gets in the bargaining. It turns out that this bargaining effect precisely cancels out the direct income effect :

Proposition 3 *Let A1-A3 hold and let A own a share η_A in B. For any $\eta_A \in [0\,1)$, A invests the same amount as if it had no participation at all.*

5 Positive results

The hold-up problem can be solved, however, when B takes an equity participation in the company it can hold up. Such participations make B internalize part of its bargaining threats, which makes its hold-up less effective. In particular :

Proposition 4 *Let A1-A4 hold*[7] *and let B own a share η_B in A. A's investment I increases in η_B, reaching the socially efficient level at $\eta_B = 1/2$.*

At $\eta_B = 1/2$, the bargaining outcome has $p = 0$. To interpret this result, it is important to remember that B's (operational) profits were normalized to zero in the setup of the model. The prediction is thus not that B will not get paid for its support, but that it won't be able to extract more than its cost, leaving A all its quasi-rents.

Notice also that at $\eta_B = 1/2$, hold-up is completely ineffective : even if B were to try it, it would not gain anything. It is tempting to interpret this result as 'by giving B ex-ante what he would get anyways, we eliminate the threat of hold-up altogether'. This interpretation works only for this particular level of participation, however.

[7]The proposition would still hold without A4, if it stated that 'the set of maximizers increases in the strong set-order'. The same is true for the other propositions that assume A4.

For further reference, it should also be noted that this '$\eta_A = 0$ and $\eta_B = 1/2$' solution is a particular instance of a class of cross-shareholdings that give efficiency. Let $\eta_A \geq 0$ denote the equity participation of A in B and η_B analogous then we have

Proposition 5 *Let A1-A3 hold. A will make the efficient investment if*

$$\eta_A = 2 - \frac{1}{\eta_B} \qquad \textit{for} \qquad \eta_B < 1$$

If, furthermore, p is exogenously restricted to be non-negative then it suffices that

$$\eta_A \leq 2 - \frac{1}{\eta_B} \qquad \textit{for} \qquad \eta_B < 1$$

This solution, however, increases the need for capital to take the participation and heightens the control issues without having any obvious advantage.

Financial structure Before proceeding with further operational implications and extensions, it is worthwhile to take a step aside and consider the financial issues that these results raise. In particular :

- Should A's original shareholders sell a stake to B at a favorable price?
- Does it matter whether B buys a share of the existing stock or brings in new capital?
- How do such equity participations interact with e.g. the equity versus debt decision?

The last issue is rather easy : Since there is no risk of bankruptcy in this model, lenders are willing to lend any amount at the risk-free interest rate. It thus follows that there are no capital restrictions on the investments, so that it does not matter with how much capital the company starts. For simplicity, assume therefore that $D = 0$ and, letting $C = E_A + E_B$ denote the firm's own capital, that C is large enough so as to not impose any restrictions on the investment level.

This also gives the answer to the second issue above : as long as the payoffs to the original investors are the same, it does not matter whether B buys existing shares or brings in new capital. Note, however, that if B is somehow capital-restricted, it will prefer to buy existing shares, since that requires only $(1 - \eta_B)$ times as much money as acquiring the same stake by bringing in extra capital.

Finally, to answer the first issue, assume that B participates by buying existing shares and that A's original shareholders and B bargain over B's share and its price prior to the firm making its investment decision, with $\eta_B \in [0,1)$ and price offers restricted to a 'large enough' but compact[8] $\mathcal{P}$. For the bargaining game, consider an infinite-horizon alternating-offer bargaining game with probability of breakdown q, with $q \to 0$. The result is

Proposition 6 *In the unique subgame perfect equilibrium, company B acquires a share $\eta_B = 1/2$ of A's equity at a total price of*

$$P_{sh} = \frac{C + R(I_0, 0) - c(I_0)}{2}$$

where $I_0 = \operatorname{argmax}_I C + \frac{R(I,1)+R(I,0)}{2} - c(I)$.

Note that B pays for the shares what they would have been worth had it not participated and that the parties effectively split the gains from B's participation. With $I_\eta = \operatorname{argmax}_I \; C + \frac{R(I,1)+R(I,0)}{2} + \frac{\eta(R(I,1)-R(I,0))}{2(1-\eta)} - c(I)$, A's original shareholders have at the start of the game an expected payoff of :

$$\frac{C + R(I_{1/2}, 1) - c(I_{1/2})}{2} + \frac{C + R(I_0, 0) - c(I_0)}{2} - C$$

so that they are indifferent about the amount of capital they supply.

The capital that B needs to take the participation does increase in C, however. It is important to notice that (the discounted profits of) unrelated activities will show up as 'capital' in these equations. To minimize the capital that B needs to commit, it might make therefore makes sense for A to spin off the activity in which B needs to take a participation, especially if it is relatively unrelated to other activities of A.

Related to this, is the question whether A's shareholders should be willing to *give* shares to B if the latter is completely capital constrained. This is relevant when B is e.g. a set of employees: would it be profitable to give employees a share in the company so as to weaken their bargaining position. This turns out not to be the case.

[8] The proof uses the fact that

$$[0 \; \eta(C + R(I_\eta, 1) - c(I_\eta))] \subset \mathcal{P}$$

where $I_\eta = \operatorname{argmax}_I C + \frac{R(I,1)+R(I,0)}{2} + \frac{\eta_B(R(I,1)-R(I,0))}{2(1-\eta_B)} - c(I)$. Larger sets are allowed, as long as they are compact.

Proposition 7 *Let A1-A3 hold. If B is completely capital constrained, then* $\eta_B = 0$.

While these financial issues are interesting, they are rather independent of the operational implications of equity participations. Since the latter are the focus of this paper, the rest of the analysis will take η_i directly as parameters, without wondering how they come about.

6 Further implications

6.1 Specialization

The proof of proposition 4 shows how equity participations reduce the importance of outside options. This suggests that they might make a firm also more willing to specialize to the other party. To analyze this issue more formally, introduce a 'degree of specialization' $\delta \in [\underline{\delta}\ \overline{\delta}]$ which management can set prior to its choice of I but after the capital and equity structure have been determined. Let this choice carry an extra cost $c_\delta(\delta)$, so that the total cost to the company is now $c(I) + c_\delta(\delta)$. Assume that[9]

$$\frac{\partial R(I,0,\delta)}{\partial \delta} < 0 < \frac{\partial R(I,1,\delta)}{\partial \delta} \qquad \text{and} \qquad \frac{\partial^2 R(I,x,\delta)}{\partial \delta \partial I} = 0$$

The intuition is then essentially confirmed in the following

Proposition 8 *Let A1-A3 hold and let there exist a unique optimal level of* δ *for each* η_B*. A's optimal level of specialization* δ *increases in B's participation* η_B*, and reaches the efficient level at* $\eta_B = 1/2$.

An analogous argument would of course show that the firm will spend less on rent-seeking actions that improve its outside option but have no further added value.

6.2 Decisions and control issues

While this paper makes explicit abstraction of the control aspect of equity participations, there are important interactions.

First of all, the 50% equity participation partially aligns B's interests with those of A's independent investors, in the sense that B's management would choose exactly the same level of investment and specialization as the

[9]The cross-partial condition excludes the possibility of indirect effects via I.

one chosen by A's management[10]. This will reduce the importance fo teh control aspects.

On the other hand, the power imbalance that comes with the large equity stakes required to get efficiency, should be a cause of concern for minority shareholders since the interests still diverge on important points, in particular the outcome of the bargaining. If it is possible to spin off the relevant activity, a joint venture might bring some relief to this concern : by grouping the control rights of all A's original shareholders into one stake in the JV, it provides a more effective counter-weight to B's power. An alternative solution is to give B non-voting equity. This was essentially the setup in the early GM-Fisher Body relationship, as discussed in the next section.

6.3 The directionality of participations and GM-Fisher Body

One of the key empirical predictions of the analysis is the direction of the equity participation : it is the firm that can commit hold-up that takes a participation in the other. In this context, it is interesting to consider the GM-Fisher Body case.

Freeland (2000) reports that, in a first phase, GM took a 60% participation in Fisher Body and placed these shares in a trust with very strong voting restrictions. Given our earlier analysis, these facts are directly consistent with the idea that GM wanted Fisher Body to make specific investments and to specialize more towards GM: the non-voting participation was GM's guarantee not to hold-up Fisher-Body. The main risk of hold-up in the other direction, the Fisher brothers leaving the company, was temporarily solved by a five-year contractual restriction.

This set-up did not solve the hold-up risk in the other direction indefinitely, however. While there is disagreement in the literature whether Fisher Body effectively committed hold-up when these contractual provisions expired, it is clear that the potential existed[11], and that GM's concurrent actions were according to the prescriptions in section 8 of this paper : GM merged with Fisher Body while the Fisher brothers got stock in the larger GM.

[10] In fact, the $\eta_B = 1/2$ participation gives a perfect alignment of interests for all decisions (except, of course, the bargaining) that have no externality on B's outside option.

[11] Note that the argument that hold-up did not effectively occur, does not at all establish that it did not play a role in the motivation for the actions taken. In equilibrium, hold-up does not occur in our models either.

7 Unilateral dependence on multiple parties

Sometimes, a firm can be subject to hold-up by more than one party. These hold-up situations might interact to various degrees. The following analyses extend the earlier results for two simple but important cases.

7.1 Separate multiple dependencies

At one extreme is the case where the focal firm faces multiple but mutually independent hold-up problems. Consider in particular a firm A that works with a number partners $\{B_i\}_{i=1}^n$ and that makes specific investments $\{I_i\}_{i=1}^n$ towards each of these B_i. Let

- Firm A's direct revenue is $\sum_{i=1}^n R_i(I_i, x_i)$ with $x_i \in \{0, 1\}$ indicating whether or not firm B_i ends up supporting A.
- Firm A's direct cost is analogously $\sum_{i=1}^n c_i(I_i)$
- All R_i and c_i satisfy A3.
- The timing is identical to the original model, except that A now chooses simultaneously all I_i in stage II. and that it has, in stage III., one-on-one negotiations[12] with each of its n customers B_i to determine a transfer price p_i.

In this case, the fact that one cooperative relationship breaks down does not affect the productivity of the other specific investments. The results from earlier thus extend directly : to get efficiency, each of these customer-firms should have a 50% participation in the focal firm.

With two partner-firms, this comes down to a joint-venture with each owning half the equity. With more than two firms, a direct application of the efficient solution is impossible, though there exists a way out if it is possible to separate each department making a specific investment I_i from the rest of A. A typical case would be a supplier A with very specific know-how who has to build plants next to each of his customers. In this case, full efficiency can be obtained by setting up a series of joint ventures between A and his customers. Each joint venture is owned 50/50 by A and the respective customer and operates the investment I_i specific to that customer B_i.

[12]It seems that the outcome is identical to the one that would obtain if we considered a multilateral bargaining with NTU Shapley value, as below. To be confirmed.

7.2 Joint hold-up

At the other end of the spectrum is the case where a firm makes one investment I that is simultaneously specific to more than one partner (in the sense that the investment loses its value if any of these parties withdraws its cooperation). Examples include an electricity plant that locates next to a mine and an aluminium smelter, or a firm that depends completely on one customer and a set of key employees.

Consider specifically the case of one focal firm A with two partners, B and C, and modify the original set-up as follows

- A's revenues are $R(I, x_B, x_C)$ with x_i indicating i's cooperation and $R(I,1,1) > R(I,1,0) = R(I,0,1) = R(I,0,0)$.
- In stage III., the parties engage in a three-way negotiation. The outcome of this negotiation is according to the Shapley value of the λ-transfer game[13] generated by the game under consideration, with $\lambda = (1, 1 - \eta_B - \eta_C, 1 - \eta_B - \eta_C)$.

In that case, we have :

Proposition 9 *Let A1-A4 hold and let B and C own shares η_B and η_C in A. A's investment I increases in both η_B and η_C, reaching the socially efficient*[14] *level at $\eta_B = \eta_C = 1/3$.*

The proof of the proposition also implies that at $\eta_B, \eta_C > 1/3$, the prices for cooperation become negative. It therefore seems logical to assume, though the lack of underlying non-cooperative bargaining mechanism makes it difficult to prove, that with prices restricted to $p_i \geq 0$, any $\eta_B, \eta_C \geq 1/3$ would give the efficient investment.

[13]Note that this bargaining game is a NTU game : A dollar can be counted more than once in the utility functions, thanks to the equity participations. There is no consensus in the literature on the appropriate generalization of the Shapley value for NTU games, but in the current context, the one based on the λ-transfer game seems to be the most logical. For a discussion of this value see e.g. Myerson (1991). The idea is simply to apply affine transformations to the players' utility functions such as to get a TU game. It is easy to check that the λ vector, when we divide each agent's utility by the corresponding component, does indeed achieve this.

[14]Note again that, although the bargaining game is a NTU game, the overall game has transferable utility if we assume risk-neutral independent investors.

8 Bilateral dependency and mergers

The analysis until now considered one firm depending unilaterally on one or more others. Another important case is that of two firms depending on each other : each partner has to make relationship-specific investments but is subject to hold-up by the other. To be more precise, let firm $i \in \{A, B\}$ have production function $R_i(I_i, x)$ with $x \in \{0, 1\}$ denoting whether cooperation eventually obtains. Firm i makes investment I_i at cost $c_i(I_i)$, and has an equity participation η_i in firm j. Let the timeline be identical to the original one, except that in each stage both firms move simultaneously.

The result of section 4 extends directly to this case and implies that a one-sided participation won't give full efficiency. On the other hand, the mechanism suggests that the effects of two participations might partially cancel each other out. Indeed :

Proposition 10 *Let A1-A3 hold for both firm A and B. The only equity participation that can give efficient investments at both sides is $\eta_A = \eta_B = 1$.*

In particular, if $\eta_A = 2 - \frac{1}{\eta_B}$ then B makes the efficient investment, and as $\eta_B \to 1$, A's investment converges to the efficient level.

The reason for stating the proposition in limit terms is the fact that the bargaining solution at $\eta_A = \eta_B = 1$ becomes degenerate : both players are indifferent about p. It follows that implementing the limit solution does not necessarily gives efficiency. Moreover, for values close to the limit, the investment incentives become extremely sensitive to the exact specification. A small deviation from the $\eta_A = 2 - \frac{1}{\eta_B}$ rule can give very suboptimal incentives. Overall, trying to implement this solution literally does not seem to be very practical.

The following subsections describe two possible solutions to this bilateral dependence problem.

8.1 Solving hold-up by internalizing activities

In the limit, the scheme of proposition 10 has $p = 0$ and complete profit-sharing between the two firms. It follows that it can be implemented de facto by merging the two firms and rewarding both managers on the basis of the overall firm profits. Since the transfer price does not play any role any more, it can be dropped. The way to implement the profit-sharing is simply by giving employees stock in the firm[15], which is indeed a widely

[15] Note that managers do not bear any personal cost in this model. Giving them one share is thus sufficient.

observed practice. Note also that this is precisely what GM did when the contract-provision that prevented hold-up by the Fisher brothers expired : they gave the Fisher brothers stock in GM and merged Fisher Body.

Implementing the same mechanism by writing contracts runs into the problem that each firm has an incentive to secretly side-contract with its own manager to cancel out the other firm's stock performance, so as to increase its bargaining power.

The merger solution generates its own problems, however. For one thing, it implies very weak incentives for private effort. Holström and Tirole (1991) also mention the extra exposure to risk as a drawback of profit-sharing. Finally, bringing the two managers in the same firm often makes them competitors for promotion, which might be detrimental for their cooperation incentives (since they might reason now on a comparative basis).

Comparison with GHM Notice that, except for the cost of investments being borne by the firm rather than by individuals, this bilateral dependency case is precisely the situation considered by Hart (1995). The essential differences are that

- The GHM models are concerned with whether owners-entrepreneurs expend the right level of effort in acquiring human capital, while the current model asks whether managers take the right decisions in making capital investments and other investments costly to the firm.

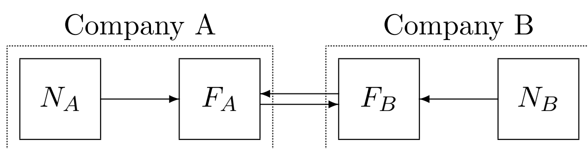
- The GHM models consider (as design variable) shifts in property rights, while the current model considers essentially shifts in the right to set incentives. This model is therefore closer to Holmström (1999).

8.2 Joint venture

Sometimes a joint venture, instead of a full merger, suffices to solve the bilateral hold-up problem. Consider the situation where the two activities making the specific investments can be separated from the rest of their respective firms. In particular, let each firm $i \in \{A, B\}$ consist of a focal and a non-focal division, denoted F_i and N_i. The non-focal division generates a fixed profit Π_{N_i}. The focal division, on the other hand, makes an investment I_i at a cost $c_i(I_i)$ which gives it a direct revenue $R_{F_i}(I_i, x_{N_i}, x_{F_j})$, with $x_k \in \{0, 1\}$ as always denoting division k's support to division F_i. Let each party's support be essential in the sense that $R_{F_i}(I_i, 1, 1) > R_{F_i}(I_i, 1, 0) =$

$R_{F_i}(I_i, 0, 1) = R_{F_i}(I_i, 0, 0)$ and let furthermore A1-A3 hold. The game is as before, again with parties acting simulatenously in each stage, and the bargaining being multilateral with the appropriate NTU Shapley value.

As in the subsection above, the inefficiencies can be solved by a full-fledged merger. As mentioned there, however, this solution can have serious disadvantages, in particular when N_A and N_B are very large divisions. Consider now the following alternative : companies A and B spin off their respective F_i divisions and put them in a joint venture which is 50/50 owned by N_A and N_B. As can be seen form figure 1 below, this transforms the bilateral dependency problem into one of a double unilateral dependency of the type discussed in section 7, for which this equity participation structure gives indeed efficiency.



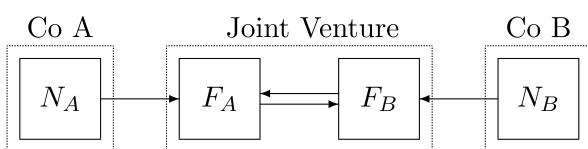


Figure 1: A joint venture can transform a bilateral dependence in two unilateral dependence relationships

9 Joint ventures

At various points in the analysis, the equity joint venture came up as a possible remedy for problems associated with simple equity participations. This section serves mainly to summarize these results.

In the case of unilateral dependencies, equity joint ventures could be useful for the following reasons :

- The discounted cash flows of unrelated activities show up as capital in the analysis of this paper. It follows that they increase the capital needed to take a 50% participation. A possible remedy is to spin-off the focal activity. The latter then becomes a joint venture between the two original firms, each owning 50% of the equity.
- With 50% of the equity, the participating firm might have too much control. The spin-off solution mentioned above would create two equal voting blocks, giving a balance of power.

A second instance where the joint venture proved useful was the case of multiple separate hold-up problems. The typical example was a firm making specific investments (e.g. co-located plants) for each of its customers. Putting the investing activities in 50-50 joint ventures between the focal firm and each of its customers could solve the hold-up problems (if such split made organizational sense).

Finally, for the case of bilateral dependence, section 8.2 demonstrated how a joint-venture could turn this problematic structure into a much easier to handle multiple-dependence structure. Notice that this transformation also minimizes the capital requirements for the participations and aligns the interests of both parents.

10 Conclusion

This paper demonstrated how equity participations can play a crucial role in solving the hold-up problem. The underlying mechanism is a change in bargaining position : a firm that holds an equity participation in the firm it is bargaining with, will take a softer stance. This effect makes hold-up less effective, in the process improving investment incentives of the other firm. The joint venture came up at different occasions as a solution to problems that arise with equity participations. The analysis also showed why mergers are the logical solution to bilateral dependence.

Firm networks and linkages seem to be a very promising research area. While there is generally little economic research on this issue, the control aspects of interfirm linkages seem to be especially interesting.

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Appendix A Proofs

A.1 Managerial objectives and altruistic bargaining

Proposition 1 *As the probability of breakdown q converges to zero, the unique subgame perfect outcome of the game converges to the Nash solution defined in (1).*

Proof :
We will follow the setup of Osborne and Rubinstein [1990] section 4.2.
Let $\underline{p}$ and $\overline{p}$ be defined

$$\begin{aligned}
\pi_A(1) + \eta_A \pi_B(1) - (1-\eta_A)\overline{p} &= \pi_A(0) + \eta_A \pi_B(0) \\
\pi_B(1) + \eta_B \pi_A(1) + (1-\eta_B)\underline{p} &= \pi_B(0) + \eta_B \pi_A(0)
\end{aligned}$$

and

$$\begin{aligned}
x_A &= \frac{\overline{p} - p}{\overline{p} - \underline{p}} \\
x_B &= \frac{p - \underline{p}}{\overline{p} - \underline{p}}
\end{aligned}$$

It then follows that $\underline{p} \le 0 \le \overline{p}$ so that the restriction $\underline{p} \le p \le \overline{p}$ gives $x_A, x_B \ge 0$. Moreover, $x_A + x_B = 1$. So the set of agreements can be defined[16]

$$X = \{(x_A, x_B) \mid x_A, x_B \ge 0, x_A + x_B = 1\}$$

Define further the utilities

$$\begin{aligned}
\hat{u}_A &= \pi_A(1) + \eta_A \pi_B(1) + (1-\eta_A)[(\overline{p} - \underline{p})x_A - \overline{p}] \\
\hat{u}_B &= \pi_B(1) + \eta_B \pi_A(1) + (1-\eta_B)[(\overline{p} - \underline{p})x_B + \underline{p}]
\end{aligned}$$

and the breakdown point $B = (0, 0)$. Note that, substituting the above definitions, these are indeed the utilities and the outside option (after breakdown) of the model we considered.
With these definition, the setup and utilities satisfy all conditions for propositions 4.1 and 4.2 in Osborne and Rubinstein [1990]. This proves the proposition.

■

Proposition 2 *As the agents impatience $(1-\delta)$ converges to zero, then the unique subgame perfect outcome of the game converges to the Nash solution defined in (1).*

[16] Note that, as argued in Osborne and Rubinstein [1990], the set of permissible offers can be larger. Since no equilibrium strategies will use these extra offers, there is no loss in generality from excluding them from the set of possible agreements.

Proof : The proof follows now the setup of Osborne and Rubinstein [1990] section 4.4.

Let the set of physical agreements X be defined as in the proof of proposition 1 with the disagreement point now equal to the earlier breakdown point $D = B$. Let the players' utilities be the δ-discounted versions of the utilities defined in the proof of proposition 1, but now normalized so that a player is indifferent between getting D today or tomorrow : agent A's utility from some agreement (x_A, x_B) at time t is $\delta^t[\hat{u}_A(x_A) - \hat{u}_A(0)]$.

With these definitions, all conditions for propositions 4.4 and 4.5 in Osborne and Rubinstein [1990] are satisfied. This proves the proposition.

■

A.2 The wrong solution

Remember that C denotes the own capital of the firm; $R(I, x)$ is the revenue of firm A with I denoting the level of investment and $x \in \{0,\ 1\}$ whether or not the firms end up cooperating; $c(I)$ is the cost to firm A of investment I; p is the agreed-upon price for B's support or participation.

Proposition 3 *Let A1-A3 hold and let A own a share η_A in B. For any $\eta_A \in [01)$, A invests the same amount as if it had no participation at all.*

Proof : Nash bargaining obviously solves

$$\max_p [(C + R(I, 1) - p - c(I) + \eta_A p) - (C + R(I, 0) - c(I))][p]$$

Note that we include the (sunk) cost of the investments. Since these costs were borne by the firm, the investments affected the working capital and are therefore still present in the objective function. As demonstrated in Van den Steen (2000), this does make an essential difference in the presence of bankruptcy. In the current context, it is easy to verify that taking the shortcut of leaving out these sunk costs in the objective function does not affect the results.

This objective function can be rewritten

$$\max_p [R(I, 1) - (1 - \eta_A)p - R(I, 0)]p$$

which gives

$$p = \frac{R(I, 1) - R(I, 0)}{2(1 - \eta_A)}$$

so that A's optimization problem

$$\max_I C + R(I, 1) - (1 - \eta_A)p - c(I)$$

becomes

$$\max_I C + \frac{R(I,1) + R(I,0)}{2} - c(I)$$

which is obviously the same optimization problem as when A would not own any stake in B.

■

A.3 Basic positive results

Proposition 4 *Let A1-A4 hold and let B own a share η_B in A. A's investment I increases in η_B, reaching the socially efficient level at $\eta = 1/2$.*

Proof : With p being the price negotiated for B's support, A's value at the end of the game is $C + R(I,x) - c(I) - xp$, where $x \in \{0, 1\}$ indicates whether or not cooperation occurs. The outside options (when bargaining on the price for cooperation) for A and B are thus respectively $(C + R(I,0) - c(I))$ and $\eta_B(C + R(I,0) - c(I))$. Nash bargaining solves

$$\begin{aligned}&\max_p [(C + R(I,1) - p - c(I)) - (C + R(I,0) - c(I))].\\ &\quad [p + \eta_B(C + R(I,1) - p - c(I)) - \eta_B(C + R(I,0) - c(I))]\end{aligned}$$

or

$$\max_p [R(I,1) - R(I,0) - p][(1-\eta_B)p + \eta_B(R(I,1) - R(I,0))]$$

which gives

$$p = \frac{(1-2\eta_B)(R(I,1) - R(I,0))}{2(1-\eta_B)}$$

Company A's expected value (at the time it has to decide on the investment) is thus

$$\begin{aligned}\hat{u}_A &= C + R(I,1) - \frac{(1-2\eta_B)(R(I,1) - R(I,0))}{2(1-\eta_B)} - c(I)\\ &= C + \frac{R(I,1) + R(I,0)}{2} + \frac{\eta_B(R(I,1) - R(I,0))}{2(1-\eta_B)} - c(I)\end{aligned}$$

which A's managers try to maximize with their choice of I.

Note that the cross partial of this expression is

$$\frac{\partial^2 \hat{u}_A}{\partial I \partial \eta_B} = \frac{1}{(1-\eta_B)^2} \frac{\frac{\partial R(I,1)}{\partial I} - \frac{\partial R(I,0)}{\partial I}}{2}$$

so that, by monotone comparative statics and A3, the choice of I increases in η_B. When $\eta_B = 1/2$, then A's objective function is

$$C + R(I,1) - c(I)$$

which gives the efficient investment.

■

Proposition 5 *Let A1-A3 hold. A will make the efficient investment iff*

$$\eta_A \leq 2 - \frac{1}{\eta_B} \qquad \textit{for} \qquad \eta_B < 1$$

with equality when negative prices are allowed.

Proof : Let A maximize $\pi_A + \eta_A \pi_B$ and B maximize $\pi_B + \eta_B \pi_A$. The Nash maximization problem then becomes

$$\begin{aligned}
&\max_p \left[(C + R(I,1) - p - c(I)) + \eta_A p - (C + R(I,0) - c(I)) - \eta_A 0\right] \cdot \\
&\qquad \left[p + \eta_B (C + R(I,1) - p - c(I)) - 0 - \eta_B (C + R(I,0) - c(I))\right] \\
&= \\
&\max_p \left[(R(I,1) - R(I,0)) - (1-\eta_A)p\right] \cdot \\
&\qquad \left[(1-\eta_B)p + \eta_B (R(I,1) - R(I,0))\right]
\end{aligned}$$

The FOC(p) is then

$$\begin{aligned}
&(1-\eta_B)\left[(R(I,1) - R(I,0)) - (1-\eta_A)p\right] \\
&\quad = (1-\eta_A)\left[(1-\eta_B)p + \eta_B (R(I,1) - R(I,0))\right]
\end{aligned}$$

or

$$2(1-\eta_B)(1-\eta_A)p = \left[(1-\eta_B) - \eta_B(1-\eta_A)\right](R(I,1) - R(I,0))$$

or

$$p = \left[\frac{1}{2(1-\eta_A)} - \frac{\eta_B}{2(1-\eta_B)}\right](R(I,1) - R(I,0))$$

Since A's payoff is $C+R(I,1)-p-c(I)$, we get efficiency if and only if p is constant in I. This will be the case when

$$\left[\frac{1}{2(1-\eta_A)} - \frac{\eta_B}{2(1-\eta_B)}\right] \leq 0$$

since in case of inequality, p will hit its 0-boundary.
This inequality can be written

$$1-\eta_A \geq \frac{1-\eta_B}{\eta_B} \geq \frac{1}{\eta_B} - 1$$

which gives then finally the result in the proposition.

■

Proposition 6 *In the unique subgame perfect equilibrium, company B acquires a share $\eta_B = 1/2$ of A's equity at a total price of*

$$P_{sh} = \frac{C + R(I_0, 0) - c(I_0)}{2}$$

where $I_\eta = \text{argmax}_I\, C + \frac{R(I,1)+R(I,0)}{2} + \frac{\eta(R(I,1)-R(I,0))}{2(1-\eta)} - c(I)$

Proof : The game satisfies the conditions of BRW (1986). It thus follows that the agreement will be Pareto-efficient, which implies $\eta_B = 1/2$ by the earlier results. To determine now P_{sh}, it suffices to calculate the Nash solution. The players outside options are

$$\begin{aligned} \underline{u}_A &= C + R(I_0, 0) - c(I_0) \\ \underline{u}_B &= 0 \end{aligned}$$

While their payoffs when they agree on a total price P_{sh} are :

$$\begin{aligned} \hat{u}_A &= \frac{1}{2}(C + R(I_{1/2}, 1) - c(I_{1/2})) + P_{sh} \\ \hat{u}_B &= \frac{1}{2}(C + R(I_{1/2}, 1) - c(I_{1/2})) - P_{sh} \end{aligned}$$

The Nash solutionthus gives that B pays

$$P_{sh} = \frac{C + R(I_0, 0) - c(I_0)}{2}$$

■

Proposition 7 *Let A1-A3 hold. If B is completely capital constrained, then* $\eta_B = 0$.

Proof : This follows immediately from the envelope theorem on the payoff to A's original shareholders :

$$(1-\eta_B)\hat{u}_A = (1-\eta_B)(C + \frac{R(I,1)+R(I,0)}{2} - c(I)) + \frac{\eta_B(R(I,1)-R(I,0))}{2}$$

so that

$$\begin{aligned}\frac{\partial(1-\eta_B)\hat{u}_A}{\partial\eta_B} &= -(C + \frac{R(I,1)+R(I,0)}{2} - c(I)) + \frac{(R(I,1)-R(I,0))}{2} \\ &= -(C + R(I,0) - c(I))\end{aligned}$$

which is negative over the relevant range by assumption 3. This proves the result.

■

A.4 Further implications

Proposition 8 *Let A1-A3 hold. A's level of specialization* δ *increases in B's participation* η_B*, and reaches the efficient level at* $\eta_B = 1/2$.

Proof : Remember from the proof of proposition 4 that, for a given η_B, A's payoff is

$$\begin{aligned}\hat{u}_A = \; & C + \frac{R(I,1,\delta)+R(I,0,\delta)}{2} \\ & + \frac{\eta_B(R(I,1,\delta)-R(I,0,\delta))}{2(1-\eta_B)} - c(I,\delta)\end{aligned}$$

The (δ,η_B)-cross-partial is

$$\frac{\partial^2\hat{u}_A}{\partial\delta\partial\eta_B} = \frac{1}{2(1-\eta_B)^2}\left(\frac{\partial R(I,1,\delta)}{\partial\delta} - \frac{\partial R(I,0,\delta)}{\partial\delta}\right)$$

which is obviously positive. Monotone comparative statics imply that the choice of δ increases in η_B. From the $\hat{u}_A$ expression, it is also easy to see that that choice will be efficient at $\eta_B = 1/2$.

■

A.5 Variations in number and kind of dependencies

Proposition 9 *Let A1-A4 hold and let B and C own shares η_B and η_C in A. A's investment I increases in both η_B and η_C, reaching the socially efficient level at $\eta_B = \eta_C = 1/3$.*

Proof : To transform this into a TU-game, notice that in the original game, any unit of income transferred from A to B gives A a loss of one, B a gain of (1-η_B) and C a loss of -η_C. It thus follows that the TU-transformation of the game is obtained by dividing B's and C's utility by a factor (1-η_B-η_C).
Introduce now the notation

$$\begin{aligned}
\alpha_B &= \frac{\eta_B}{1-\eta_B-\eta_C} \\
\alpha_C &= \frac{\eta_C}{1-\eta_B-\eta_C}
\end{aligned}$$

Applying now the Shapley value gives A a payoff :

$$\begin{aligned}
&\frac{1}{3}\left[(1+\alpha_B+\alpha_C)(R(I,1,1)-c(I))-(\alpha_B+\alpha_C)(R(I,0,0)-c(I))\right] \\
&\quad +\frac{1}{6}\left[(1+\alpha_B)(R(I,1,0)-c(I))-\alpha_B(R(I,0,0)-c(I))\right] \\
&\quad +\frac{1}{6}\left[(1+\alpha_C)(R(I,0,1)-c(I))-\alpha_C(R(I,0,0)-c(I))\right] \\
&\quad +\frac{1}{3}\left[(R(I,0,0)-c(I))\right] \\
&= \frac{1}{3}\left[R(I,1,1)+(\alpha_B+\alpha_C)(R(I,1,1)-R(I,0,0))\right] \\
&\quad +\frac{1}{6}\left[R(I,1,0)+\alpha_B(R(I,1,0)-R(I,0,0)\right] \\
&\quad +\frac{1}{6}\left[R(I,0,1)+\alpha_C(R(I,0,1)-R(I,0,0))\right] \\
&\quad +\frac{1}{3}\left[R(I,0,0)\right]-c(I)
\end{aligned}$$

with $R(I,0,1) = R(I,1,0) = R(I,0,0)$, this becomes

$$\frac{R(I,1,1)}{3} + \frac{\alpha_B+\alpha_C}{3}\left[R(I,1,1)-R(I,0,0)\right] + \frac{2R(I,0,0)}{3} - c(I)$$

so that investments clearly increase in both participations and become efficient at $\alpha_B = \alpha_C = 1$. The latter gives $\eta_B = \eta_C = 1/3$. Note that other combinations of α_B and α_C would work too, but they would always involve a larger total participation, negative prices and different participations for both players (which gives control issues).

■

Proposition 10 *Let A1-A4 hold. The only equity participation that can give efficient investments at both sides is* $\eta_A = \eta_B = 1$.

In particular, if $\eta_A = 2 - \frac{1}{\eta_B}$ *then* B *makes the efficient investment, and as* $\eta_B \to 1$, *A's investment converges to the efficient level.*

Proof :

Note that by proposition 5, efficiency on the part of A requires :

$$\eta_A \leq 2 - \frac{1}{\eta_B}$$

and analogous for B. Some algebra shows, however, that these inequalities can only hold simultaneously only when $\eta_A = \eta_B = 1$. This gives the first part of the proposition.

The second part follows from the if-part of proposition 5, applied twice, the fact that in the limit both equalities hold, and the theorem of the maximum for a unique optimizer.

■