Interest Rate Stabilization vs. Monetary Control: Another Reconciliation

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Abstract

This paper presents a reconciliation of the real-bills doctrine with the notion of control of monetary aggregates. We consider an overlapping generations model with banks that trade currency on a competitive money market. Because of market incompleteness banks underestimate the social value of a unit of cash reserves and tend to deplete their private reserves too fast. The consequence of this is excessive interest rate spikes on the money market in periods of scarce liquidity. The central bank can intervene, in the spirit of the real-bills doctrine, injecting funds to stabilize the interest rate. In doing that, though, the central bank faces a form of market mediated moral hazard, banks tend to deplete their reserves even more and the final effect is an increase in nominal expenditure and nominal instability. The introduction of a legal reserve requirement in order to control bank credit expansion is rationalized as it helps to reduce the nominal variability associated with interest rate stabilization.

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1 Introduction

The adequate supply of means of payment to the economy is a classic theme in the literature on central banking. The real-bills doctrine advocates for adjustments of the stock of money in order to meet the "needs of trade". In practice, these adjustments are geared towards stabilizing the interest rate on the interbank market. The original purpose of the Federal Reserve Act was precisely to supply "an elastic currency" to guarantee a smooth functioning of the payment system[1]. A rich empirical literature has documented how the creation of the Fed virtually eliminated seasonal fluctuations in interest rates and the spikes in interest rates which were associated with liquidity crises before 1914[2]. More recently, the preference for interest rate targeting as an operating procedure and the tendency to 'smooth' the interest rates path by the Federal Reserve have been attributed —to some extent— to a similar concern for the regular functioning of the payment system. On the other hand, monetarist thinking has accustomed us to the idea that there is a need for control of the monetary aggregates, that is for a set of regulatory requirements and for a tight monitoring of the banks' balance sheets in order to keep under control the total supply of means of payment in the economy. In the absence of such restrictions the economy would be subject to large shocks to the money multiplier, causing unwanted nominal instability.

In existing models of the payment system it is hard to reconcile these two ideas. If an elastic supply of currency is needed, why restrict banks from offering currency substitutes when money is scarce? Sargent and Wallace (1982) article takes this line of reasoning to its extreme consequences. In their model the banking system can offer a perfect substitute to money, and if it is not restricted in this activity it can provide on its own an elastic supply of means of payments, without the need of a central bank. The need for an elastic supply of currency arises in their model as a consequence of a legally imposed reserve requirement that limits the ability of the banks to freely supply money substitutes. Thus, an elastic supply of currency can be achieved either by central bank intervention or by eliminating the regulatory restriction altogether. In this approach, which we dub the 'pure' real-bills doctrine, any proposition supporting interest rate stabilization can be reinterpreted as a proposition in favor of free banknotes issue[3]. Therefore, while

[1] Quoted from the preamble to the Federal Reserve Act.

[2] See Miron (1986), Mankiw, Miron and Weil (1987), Goodfriend and King (1988).

[3] Champ, Smith and Williamson (1996) make the parallel explicitely, interpreting their model both as a representation of the Canadian experience with free banking and as a

providing a rationale for interest rate stabilization, this approach does not justify a concern with the loss of control of monetary aggregates because of banks' activity. Most existing monetary systems, though, are characterized both by regulatory control on the creation of money substitutes and by some form of interest rate stabilization by central banks.

The goal of this paper is to reconcile interest rate stabilization by the central bank with the notion of monetary control. A model is presented where the same underlying imperfection accounts both for the need of currency injections in times of scarce liquidity *and* for the need of a reserve requirement. In the model presented the introduction of a reserve requirement allows to achieve interest rate stabilization with a smaller level of nominal instability.

The monetary framework of the model is given by a simple overlapping generation structure with a transactional demand for money. Banks provide contingent liquidity as in Diamond and Dybvig (1982). In order to introduce an active money market we consider a model with banks active in different regions, affected by different liquidity shocks, which trade money ex post to satisfy the liquidity needs of their customers. The crucial imperfection in the money market is the lack of insurance against bank-specific liquidity shocks.

We can describe briefly the main mechanism at work. Banks need to keep reserves in order to satisfy their depositors' liquidity demand. They can either hold reserves on their own or borrow from other banks when they need them. Because of market incompleteness they undervalue the gains from having extra reserves to lend in states of scarce aggregate liquidity, and they tend to hold too little reserves per unit of deposit. In section 3 we discuss in detail the pecuniary externality underlying this insufficient supply of reserves. Because of this imperfection reserves acquire a 'public good' status, and banks on aggregate tend to hold too little of them in the states when they are more badly needed. This, generates excessive spikes in the interest rate in states of scarce liquidity. A problem of this type was discussed informally by O. M. W. Sprague —a prominent advocate of the creation of the Fed— in his book on the National Banking System. The central bank can partially correct this state of affair by lending reserves in states of scarce liquidity. However in this way it has to expand total money supply, generating some nominal instability and accomodating the tendency of private banks to expand nominal credit on too little reserves. Central supply of reserves incurs in a form of market mediated moral hazard. In this situation, a reserve requirement can make the policy of interest rate

representation of an economy with elastic currency supplied by a central bank.

stabilization more effective.

This paper attempts to build a bridge between a problem of monetary policy and a problem of banks' regulation. Therefore, it is related to two quite distinct strands of literature. In the monetary literature we have already mentioned Sargent and Wallace (1982), and we will often make reference to it. Freeman (1996) presents a sharp case for interest rate stabilization in the payment system. Moreover, given the timing assumptions in his model, monetary policy can achieve the first best allocation by stabilizing the interest rate without affecting the nominal price level. In our paper instead, some nominal instability is generated by central bank intervention. Moreover, unlike in Sargent and Wallace (1982), nominal instability will have negative welfare consequences and this will make undesirable a complete smoothing of the interest rate. Woodford (1997) constructs a quasi-representative agent setup with cash-in-advance and a sequential service constraint where the distribution of liquid resources across units hit by different shocks is central. Woodford model is very much related to the model presented here, but in the quasi-representative-agent setup he adopts wealth effects have no welfare consequences, so that the constrained efficiency issues studied in the present paper do not arise.

On the other hand, this paper owes much to the literature on banking and liquidity provision, dating back to Diamond and Dybvig (1983). In particular, a multi-region extension of that model was introduced in Battacharya and Gale (1986). Battacharya and Gale study the problem of the optimal design of an incentive-compatible mechanism to reallocate liquid resources *ex post* assuming that the central bank offers commercial banks a contingent liquidity contract analogous to that used between consumers and banks. In a sense, in Battacharya and Gale all interbank transactions must be mediated by the central bank, who monitors perfectly the trades made by each individual bank. In this paper instead an anonymous market mechanism is used to reallocate funds across banks located in different regions, and central bank intervention takes the more conventional form of open market operations. The introduction of a competitive financial market to reallocate resources between banks in a multi-region setup appears in a recent paper by Allen and Gale (2000). Their paper discusses the general problem of integrating financial markets and intermediaries in a non-monetary model of liquidity provision. Our proposition 3 is closely related to the constrained efficiency analysis presented in that paper, which in turns is related to the classical result by Geanakoplos and Polemarchakis, in the incomplete markets literature. Some other recent articles, in particular Caballero and Krishnamurty (2000) and Holmstrom and Tirole (2000), have shown how constrained inefficiency arises in models of liquidity provision in the absence of a grand intermediation contract that provides liquidity insurance to the whole economy. Banerjee and Maskin (2000) study the interaction between government regulation of financial intermediaries and monetary stability. Their focus is rather distinct, since they look at the risk shifting effects of banks'limited liability of banks and of deposit insurance, while our attention is on the functioning of the money market. It is interesting to observe that they reach similar conclusions on the validity of a two-pronged approach, that couples a reserve requirement with a policy of reserves' injections in periods of scarce liquidity.

The paper is organized as follows. Section 2 presents the basic assumptions of the model. In section 3 I characterize the stationary equilibrium and derive a result of excess interest rate volatility and insufficient reserves. In section 5 I discuss monetary policy and reserve requirements. Section 6 concludes.

2 The model

The environment

Consider an exchange economy with overlapping generations of households that live three periods. Each generation is made of a continuum of households located in n different regions. Each region population is normalized to 1. Each household born at date $t-1$ is composed of a producer and a consumer, the producer receives an endowment of x units of consumption good when young (date $t-1$) and y units when middle aged (date t), the consumer consumes when middle aged and old. The consumption good is non-storable and consumers cannot consume the good produced in house, and need to exchange their whole endowment. Household preferences are represented by the utility function[4]

$$\theta_t u(c_t) + (1-\theta_t)u(c_t + c_{t+1})$$

where u is a strictly concave function with a coefficient of relative risk aversion greater than 1..θ_t is an unobservable preference shock, realized at date t, which takes the values 0 or 1. All uncertainty regarding generation t is resolved at date $t+1$, at this date in region j a proportion β^j of consumers receives the shock $\theta = 1$, i.e. they are revealed to be early consumers. Early

[4] To save on notation we omit individual subscripts. We will introduce regional subscripts in a moment and all consumers of the same type in a given region will have equal treatment.

consumers must consume when middle aged, while late consumers have the choice to consume when middle aged or old. At date t each region j is assigned a random β^k drawn from the distribution F_o in odd periods and from the distribution F_e in even periods. There is no aggregate uncertainty: both distributions F_o and F_e are discrete and the cross sectional distributions of the shocks correspond to the ex ante distributions. The economy-wide proportion of early consumers appearing in the economy in odd and even periods are $\overline{\beta}$ and $\underline{\beta}$. We assume

$$\underline{\beta}(x + y) < x < \overline{\beta}(x + y). \tag{1}$$

This inequality characterizes odd periods as periods of scarce liquidity.

Money, banks and markets

There is a fixed supply of fiat money m in the economy. In an OLG setup money has a role to help intergenerational exchange, in addition, we allow for a transactional demand for money by introducing a spatial friction. Consumers operate on two 'islands', the first is called the 'money market island', the second the 'goods market island'. The goods market is a cash market: on the goods market island the only type of trades admitted are spot trades of cash for goods[5]. On the other hand, on the money market island intertemporal exchanges are admitted, and they take two forms: banking contracts (banks) among consumers located in the same region and interbank lending between banks located in different regions. The money market island is divided in n regions. All consumers born at time t in region j form a banking contract, a bank. Consumers in a bank pool the cash receipts from the sale of their endowments in their first two periods of life. Then, the banking contract specifies contingent transfers of cash at dates $t+1$ and $t+2$, we will be more precise on the form of the banking contracts in the following paragraphs. Banks operating in different regions can reallocate cash among themselves at date $t + 1$ by borrowing and lending on the money market.

Let now describe in detail the life of a typical group of households born in region j at time t. For clarity, let divide each time period in 4 stages. At time t, stage I, the producers travel to the goods market and sell x for cash in stage III. In stage IV they come back with $p_t x$ and deposit it in the bank. At time $t + 1$ consumers are revealed to be early consumers in two stages.

[5]On the goods market island there is no technology to sign contracts with a non-falsifiable signature, because all notaries are located on the money market island. Therefore, the only asset that can be used for payments is the outside liquid asset: money.

In stage I a fixed fraction α of consumers is revealed to be early and have to leave the money market island immediately, receiving d from their bank. They will be called 'early leavers'. Producers leave the money market as well. Then, in stage II, an additional fraction $\beta_j - \alpha$ are revealed to be early consumers. The early consumers that are not early leavers and, possibly, late consumers, receive cash from the bank and leave for the goods market island. In stage III consumers and producers exchange goods and cash on the goods market island. In stage IV producers return to the money market island and deposit their receipts $p_{t+1}y$. At time $t+2$, first of all banks clear with each other their financial obligations, and then transfer the remaining cash to the late consumers that travel to the goods market island and make their purchases. The timeline of a typical household's life is summarized in Table 1.

Now we can be more precise about the banking contract. A banking contract is a 4-uple $(d, c_{11}, c_{21}, c_{22})$, where d is the fixed amount of cash transferred to early consumers leaving in stage I, c_{11} is the consumption level of early consumers leaving in stage II, c_{21} and c_{22} are the consumption levels of late consumers at date $t+1$ and $t+2$. c_{11}, c_{21} and c_{22} are functions of the regional shock β_j, and possibly of the interest rate and of the price level[6].

Consumers located in region j underwrite a banking contract at date t, and they use it in the following dates to finance their consumption selecting c_{11} or (c_{21}, c_{22}). Notice that, even though individual preference shocks are unobservable the contract above is incentive compatible provided that it satisfies the following inequalities

$$\begin{aligned} c_{11} &\geq c_{21} \\ c_{22} &\geq c_{21} + c_{11}. \end{aligned}$$

Notice that consumers serviced by a bank contract do not need to participate in the financial market directly, since the banks do that on their behalf. Actually, it is the very fact that banks can limit consumers' participation that allows them to offer liquidity insurance and to improve upon the financial market allocation. If consumers can do side trades on the financial market the contracts of contingent liquidity supplied by banks become redundant as was first shown by Jacklin (1986).

The following assumption limits the ability of households to form a big interregional banking arrangement. Under this assumption interregional

[6] Since we will study equilibria with non-stochastic price sequences this will not be needed.

banking arrangements that improve upon the financial market allocation are not feasible.

Assumption 1 *Households located in region j cannot observe the level of consumption nor the trades on the financial market of households located in other regions.*

If a bank in region j were to offer a banking contract to a group of consumers located in a different region the incentive compatibility constraint would now take the form

$$\begin{aligned}(1+r)c_{11} &\geq (1+r)c_{21}+c_{22}\\ (1+r)c_{21}+c_{22} &\geq (1+r)c_{11}\end{aligned}$$

where r is the real interest rate. This set of incentive compatibility constraints reflects the fact that now consumers can use the cash received from the banks to operate freely on the money market. It is possible to show that a banking contract that satisfies this constraints is redundant, in the sense that consumers achieve the same level of utility offered by a banking contract by just keeping their endowments and trading *ex post* on the financial market. This is again a consequence of the problem highlighted first by Jacklin's (1986).

This assumption makes clear what is the empirical counterpart to the notion of "region" used in the model. A region can be any set of firms and consumers whose balance sheets are easier to monitor for a given bank. Notice that in this type of setup banks can improve upon financial market because they offer a commitment to lend at a rate different from the current market rate when the liquidity shock hits. To successfully implement this type of contract the bank must be able to observe the customer activity to make sure he is not using the commitment simply to channel funds to the money market when the interest rate differential is favorable (irrespectively of his consumption level in that period). That is, the bank must be able to observe the customer balance sheet and it must be able to impose some penalty on the customer in the case of misuse of the credit line. In actual intermediation contracts, the fact that a bank offers a full set of payment services and corporate finance services, helping a company to issue commercial paper, etc. helps the bank to enforce contingent liquidity arrangements. These considerations underlie the idea that the bank has a limited group of customer whose balance sheet can be monitored.

Even in the absence of a big interregional bank a better allocation of liquidity can be achieved introduction of liquidity insurance among regional banks as in Battacharya and Gale (1982). Here we rule out this type of arrangement by assuming that the bank specific shocks β_j are unobservable and that banks cannot monitor each others' balance sheet. This renders interbank contracts of liquidity insurance useless. More precisely, for the same reasons that make a cross-regional bank unfeasible, an incentive compatible contract of liquidity insurance across banks cannot improve upon the allocation achieved by the ex post financial markets. In short, the total present value of the transfers between the two banks, computed at the market price r, must be equal to zero and these same transfers can be achieved simply by *ex post* trading between the two banks. Without going into an explicit model of endogenous asset design we make directly the following assumption.

Assumption 2 *Bank specific shocks β_j are non insurable: there are no assets contingent on the individual distribution of the β_j.*

In this economy consumers located in region j form a banking contract while regional banks interact only through the ex post financial market. In short, consumers obtain liquidity through local banks and local banks obtain liquidity through the interbank market.

Equilibrium

A competitive equilibrium with banking contracts is defined as a sequence of banking contracts $(d_t, c_{11,t}, c_{21,t}, c_{22,t})$, and a sequence of interest rates i_t and price levels p_t, such that:
(i) the banking contract maximizes the expected utility at date t of consumers located in a representative region;
(ii) the goods markets and the money markets clear at each date t.

As a first step let consider the bank problems at date t

$$\begin{aligned} \max \quad & E\left[\alpha u(d/p_t) + (\beta_k - \alpha)\, u(c_{11}^k) + (1-\beta_k)u(c_{21}^k + c_{22}^k)\right] \qquad (2) \\ s.t. \quad & (1+i)\left[\alpha d + p_t\,(\beta_k - \alpha)\, c_{11}^k + p_t(1-\beta_k)c_{21}^k\right] + p_{t+1}(1-\beta_k)c_{22}^k \\ = \quad & (1+i)xp_{t-1} + yp_t \end{aligned}$$

The cash in advance constraint is not present in the individual optimization because banks they can borrow money before consumers leave for the goods market island and are price takers on the money market. The cash in advance constraint is relevant, though, at the aggregate level since the clearing of the money market requires that total expenditures of consumers born at date $t-1$ are no larger than the money earned by these consumers.

Consumers just born have no money endowment and old consumers have no reason to lend their money. Therefore, equilibrium in the money market requires

$$\alpha d + p_t \frac{1}{n} \sum \left[\left(\beta^k - \alpha \right) c_{11}^k + (1-\beta_k) c_{21}^k \right] \leq p_{t-1} x \tag{3}$$

with strict equality if $i > 0$.

A stationary (periodic) equilibrium is characterized by a pair of banking contracts, prices and interest rates, one for even and one for odd periods. With a fixed supply of fiat money m there is a unique stationary equilibrium with a costant price level $p = m/(x+y)$ and with $\underline{i} = 0 < \overline{i}$[7]. Let proceed with the characterization of this equilibrium. Banks servicing consumers born in odd periods can fully smooth consumption across all types of consumers and choose[8]:

$$\underline{d}/p = \underline{c}_{11} = \underline{c}_{21} + \underline{c}_{22} = x + y.$$

Market clearing on the money market is given by

$$\underline{\beta}(x+y) + \left(1 - \underline{\beta}\right) \underline{c}_{21} = x$$

condition (1) guarantees that $\underline{c}_{21} \geq 0$.

Consider now banks servicing consumers born in even periods, that have to supply liquidity in odd periods when liquidity is scarce. Their consumption levels are characterized by the following conditions together with the budget constraint in (2)

$$u'(\overline{c}_{11}^k) = (1+i)\, u'(\overline{c}_{22}^k) \tag{4}$$

$$\overline{c}_{21}^k = 0 \tag{5}$$

$$u'(\frac{\overline{d}}{p}) = Eu'(\overline{c}_{11}^k) \tag{6}$$

From these conditions we can derive the aggregate demand on the money market. With separability we can show easily that aggregate demand of funds is a decresing function of i. Market clearing on the money market is given by

$$\alpha \overline{d} + \frac{1}{n} \sum p \left(\overline{\beta}^k - \alpha \right) \overline{c}_{11}^k = px \tag{7}$$

and condition (1) implies that $\overline{i} > 0$.

[7] A proof of uniqueness is in the appendix.

[8] A remark on notation: we denote with an under (lower) bar prices relative to even (odd) periods and all quantities relative to consumers born in odd (even) periods.

In the following section we will study more in detail the choice to hold reserves and the welfare properties of the equilibrium described.

The equilibrium of a credit economy

An useful benchmark for the analysis to follow is the stationary competitive equilibrium of the economy without the cash in advance constraint. If consumers can buy on credit when young, the aggregate constraint (3) disappears and is replaced by the goods market equilibrium conditions. Fiat money will still be used for intergenerational exchange, but we can show that it will no longer carry a premium in liquidity scarce periods. The equilibrium in this case is characterized by full smoothing for both even-born and odd-born consumers and a zero real interest rate

$$\overline{c}_{11} = \underline{c}_{11} = \overline{c}_{12} + \overline{c}_{22} = \underline{c}_{12} + \underline{c}_{22} = x + y$$

In this economy young consumers also can participate to the financial market even if they do not have money, by simply selling their goods x for credit. The participation of young even-consumers of the next generation to the money market allows middle aged odd-consumers to borrow in aggregate, so as to spend more than x. In a stationary equilibrium the higher consumption of this early consumers is associated to a low consumption of old even-consumers. This consumers are indifferent between consuming in the liquidity scarce or in the previous liquidity abundant state, so their shift towards the liquidity abundant state frees resources for the odd-consumers.

3 Reserves and interest rate fluctuations

A useful benchmark to consider first is the case of no bank-specific risk, that is $\beta^k = \beta$. In this case the equilibrium allocation in the odd periods is given by

$$\overline{c}_{11} = \frac{x}{\overline{\beta}} < x + y < \frac{y}{1 - \overline{\beta}} = \overline{c}_{22}$$

Notice that if we remove either assumption 1 or 2 an economy with bank-specific shocks would replicate the allocation of this benchmark economy. That is, this benchmark case covers also the case of a fully intermediated economy and the case of fully insurable bank specific shocks.

If we remove bank-specific risk the stationary equilibrium is Pareto optimal despite the presence of a monetary friction and of a positive interest rate in periods of scarce liquidity.

Proposition 1 *When there is no bank-specific risk, i.e.* $\overline{\beta}^k = \overline{\beta}$*, the competitive equilibrium is Pareto optimal.*

The fact that the real rate is positive in every odd period suggests that a variant of the criterion of Kareken and Wallace holds for this economy. Given the three period structure and the presence of the cash-in-advance constraint we present a direct proof of optimality in the appendix. In the appendix we also show that the equilibrium allocation maximizes a weighted sum of the utility of consumers born in even and odd periods. We will discuss again the welfare criterion in the section on monetary policy.

Let now turn to an economy with bank-specific risk in the state of scarce liquidity. With bank-specific risk regions hit by different shocks will have different consumption levels. Since there is no aggregate risk this immediately implies that the equilibrium is not Pareto optimal from a first best perspective. In particular, given our assumptions, consumers located in regions with a high liquidity shock (apart from early leavers) will have a lower level of consumption. Thus, regions with a higher liquidity shock have a higher marginal value of money. Moreover, the banks operating in the regions characterized by scarce currency are taking lending positions on the interbank market. This two facts are established in the following lemma.

Lemma 2 *In the odd period there is an increasing relationship between the liquidity shock $\overline{\beta}^j$ and $u'(\overline{c}^j_{11})$, and an increasing relationship between $\overline{\beta}^j$ and the net borrowing position of the regional bank on the money market, $\alpha\overline{d} + p(\overline{\beta}^j - \alpha)\overline{c}^j_{11} - px$.*

In this context a reduction of the interest rate i would have a favorable effect in terms of liquidity reallocation, by transferring currency from lenders to borrowers. This consideration will provide a motive for interest rate stabilization when we introduce monetary policy in the next section. In even periods currency is abundant with respect to the existing credit demand and the absence of a liquidity premium allows all consumers to achieve the same consumption level $x + y$. In odd periods, instead, currency is scarce and the lemma above shows that the positive interest rate tends to hurt consumers in regions hit by a high liquidity shock. When the aggregate liquidity shock is high it hurts relatively more banks with a high bank-specific shock.

Since the interest rate is determined by the amount of funds available on the money market, we now turn to the decision of banks to hold reserves. In particular consider banks servicing consumers born in the even period $t-1$, which supply liquidity in the odd period t. The choice to hold reserves is relevant in stage I of period t. At this point in time a bank can decide to part with a portion of its reserves $\alpha\overline{d}$, to finance the purchases of early leavers. Let us define the level of reserves left at the end of stage I in an individual bank j

$$w = px - \alpha \overline{d} \tag{8}$$

and denote by W the aggregate level of reserves. In equilibrium, clearly, one has $W = w$. As noticed above, the total amount of reserves W available on the money market determines the equilibrium interest rate. In particular, one can define the function $\phi(W)$ that associates to every level of W (i.e. of $\overline{d}$) the nominal interest rate i that clears the money market[9]. Lemma 6 in the appendix shows that ϕ is a decreasing function. With this definitions in hand one can reconsider problem (2) from the vantage point of stage I of time t, before the bank-specific liquidity shock is realized, and define the indirect expected utility function[10]

$$\begin{aligned} V(w, W) = \quad & \alpha u(\frac{x - w/p}{\alpha}) + \max E\left[\left(\beta^k - \alpha\right) u(c_{11}^k) + (1 - \beta^k) u(c_{22}^k)\right] \\ & s.t. (1+i)\left(\beta^k - \alpha\right) c_{11}^k + (1 - \beta^k) c_{22}^k = (1+i)\frac{w}{p} + y \\ & i = \phi(W) \end{aligned}$$

Banks will offer contracts that maximizes expected consumers utility $V(w, W)$ with respect to its first argument. Therefore, the level of rerserves in a competitive equilibrium can be compactly characterized by the condition[11]

$$V_1(w, w) = 0$$

Consider now the problem of a social planner —acting on behalf of the generation born at time $t-1$— that has the power to set the level of reserves held by banks at stage I of time t, and lets the financial market determine the allocation of reserves after bank-specific shocks have realized. .That is, we assume that after date 0 the social planner is subject to the same constraints faced by the private economy: reallocation across regions can take place only through borrowing and lending (i.e. subject to incentive compatibility with non-monitorable side trades). This corresponds to the usual thought experiment made in the incomplete markets literature when discussing constrained efficiency[12]. The social planner maximizes $V(w, w)$

[9]This is the interest rate that satisfies conditions (4) to (7), fixing a certain $\underline{d}$ and omitting condition (6).

[10]From here on, we concentrate on the odd period, and we omit the upper bar where no confusion is possible.

[11]V_k denotes the partial derivative of V with respect to its k-th argument. Differentiability of V and concavity in the first argument can be proved easily in this context. Strict concavity implies also that only symmetric equilibria exist. A simple envelope argument shows that this condition is equivalent to condition (6).

[12]Geanakoplos and Polemarchakis (1986).

with respect to both arguments and the constrained efficient allocation is compactly characterized by the condition

$$V_1(w^*, w^*) + V_2(w^*, w^*) = 0.$$

The difference between the constrained efficient allocation and the competitive equilibrium depends solely on the term V_2. Applying a standard envelope argument we obtain

$$V_2 = E\left[(1+i)\, u'(c_{11}^k)\left(\frac{w}{p} - \left(\beta^k - \alpha\right) c_{11}^k\right)\right] \phi'(w) \qquad (9)$$

The essential difference between the competitive allocation and the second best allocation is that competitive banks do not take into account the effect of their choice on the *ex post* price of funds i. An immediate consequence of Lemma 1 is that the expression in (9) is positive at a competitive equilibrium. The lemma shows that the net lending position of a bank and the marginal value of money are negatively related, moreover —by market clearing— the net lending positions have zero mean ($E\left(\frac{w}{p} - \left(\beta^k - \alpha\right) c_{11}^k\right) = 0$), so the expectation term is a covariance and is negative. ϕ is a decreasing function and we conclude that the expression in (9) is positive.

This is the main step to derive the following proposition.Notice that the constrained efficiency analysis done here concerns solely the welfare of consumers born at date $t-1$. All other consumers' utility is not affected by the level of reserves available on the money market at date t. so long as the total money supply is kept fixed at m the equlibrium price sequence and the consumption of all other consumers is not affected by w, and we can safely conduct our analysis concentrating on one generation. When we introduce an active monetary policy this will no longer be the case and issues of intergenerational transfers will be unavoidable.

Proposition 3 *In odd periods the economy is constrained Pareto inefficient and banks hold an inefficiently low level of reserves $w < w^*$.*

The private banking economy is inefficiently illiquid, that is banks issue too many deposits commitments to early leavers and keep an inefficiently small reserve of the liquid asset. .In even periods, instead, it can be shown easily that the expression (9) is zero. The reserves held in even periods are optimal both from the private and from the social point of view. That is, the problem of insufficient liquid reserves tends to appear exactly in the states when aggregate liquidity is scarce. Moreover we have $\phi(w) > \phi(w^*)$ and the economy displays excessive interest fluctuations, which is a symptom of the underlying illiquidity problem.

The substantial interest of this result depends on the magnitude of the externality captured in expression (9). Therefore, this is the place to discuss under which conditions we expect this term to be significant. The first condition is that some regional risk has to be present, if there are no bank-specific shocks all banks have a zero net position on the federal funds market and the economy is constrained efficient. The volume of trades on the money market documented in the introduction indicates that this term is likely to be sizeable in actual economies. The second condition is that differences in the marginal utility of funds has to be large. If agents are risk neutral u' is constant and differences in the amount of funds available to banks hit by different shocks have no consequences on net welfare. A broad interpretation of this condition is that it captures imperfections in the credit market such that a customer that was financed by a bank j cannot easily move to a bank j with abundant liquidity. The third condition concerns the presence of withdrawals by 'early leavers', that is of funds that, because of banks' decisions, do not flow back easily into the money market, causing temporary shortages of liquidity. Here the physical structure of the model generates a demand for this type of rigid committments. More generally a characteristic of the banking business is that it offer financial services in the form of committed sources of finance (deposits and credit lines)[13]. The use of these committed sources of finance exposes banks to sudden withdrawals, and in the aggregate exposes the money market to temporary shortages of liquidity. It is outside the scope of this paper to study why this type of rigid forms of financing are present and are typical of the banking business.

4 Monetary policy and interest rate stabilization

In the previous section it was observed that banks illiquidity generates excessive interest rate fluctuations. We now introduce a monetary authority that attempts to stabilizes the interest rate by injecting currency in the system in periods of scarce aggregate liquidity. More specifically suppose that, when the money market is open in stage I of odd periods, the central bank intervenes by lending h units of fiat money. The net return from this intervention, ih, is returned to old consumers at the beginning of the following period so as to keep the money stock constant. Total expenditure

[13] See Diamond, Rajan and Stein on the supply of committed sources of finance.

is now $m+h$ in odd periods and m in even periods so that the price level will no longer be constant. As in Sargent and Wallace (1982) and unlike in Freeman (1996) a monetary policy that stabilizes the interest rate has a destabilizing effect on prices. Again we concentrate the analysis on the stationary (periodic) equilibrium. The price level in even and odd periods will be, respectively

$$\underline{p} = \frac{m}{x} < \overline{p} = \frac{m+h}{x+y}.$$

The real interest rate in the even period will still be 0, which requires a positive nominal rate in even periods:

$$1 + \underline{i} = \frac{\overline{p}}{\underline{p}} = \frac{m+h}{m}.$$

The real allocation for consumers born in odd periods will be

$$\underline{d}/p = \underline{c}_{11} = \underline{c}_{21} + \underline{c}_{22} = (1+\underline{i})\,x + \frac{1}{1+\underline{i}}y. \tag{10}$$

the condition (1) is still sufficient to guarantee that $\underline{c}_{21} \geq 0$, because these consumers sell the first part of their endowment in periods of high prices, so their monetary receipts are tilted towards the beginning of their life. Therefore currency scarcity is still not a problem for these consumers. To summarize the effect of a real-bills oriented monetary policy in even periods is to increase the nominal rate ($\underline{i} > 0$) without affecting the real rate. The effect on the utility of agents born in odd periods is ambiguous as can be seen from expression (10). In the special case $x = y$ the effect is positive, and these consumers have a smooth and higher level of consumption under this policy. Let now turn to the effects of this policy on the odd-periods money market, which is the original target of this policy.

The budget constraint for banks active in odd periods is

$$\alpha\frac{\overline{d}}{\overline{p}} + \left(\overline{\beta}^j - \alpha\right)\overline{c}^j_{11} + \frac{1}{1+\overline{r}}\left(1 - \overline{\beta}^j\right)c^j_{22} = \frac{\underline{p}}{\overline{p}}x + \frac{1}{1+\overline{r}}\left(\frac{\overline{p}}{\underline{p}}y + \overline{i}\frac{h}{\underline{p}}\right)$$

where $\overline{r}$ is the real interest rate. The market clearing condition on the money market is

$$\alpha\frac{\overline{d}}{\overline{p}} + \frac{1}{n}\sum\left(\overline{\beta}^j - \alpha\right)\overline{c}^j_{11} = \frac{\underline{p}}{\overline{p}}x + \frac{h}{\overline{p}}.$$

The increased supply of funds on the money market is associated to changes in the price levels and changes in the wealth of consumers. Some algebra shows that the first effect dominates, so that the real interest rate decreases in equilibrium. The nominal interest rate decreases *a fortiori* since expected inflation $\frac{\underline{p}}{\overline{p}}$ decreases. We can summarize the stabilizing effects of this monetary policy in the following proposition.

Proposition 4 *Suppose that the central bank intervenes by lending fiat currency $h > 0$ in odd periods and distributing the net returns $\bar{i}h$ to old consumers in even periods. The effects of this policy are: the real interest rate in even periods is unchanged and equal to 0; the nominal interest rate in even periods increases; the real and nominal interest rate in odd periods decrease.*

Thus this policy has the desired effect of reducing interest rate fluctuations, and the side effect of generating nominal instability in the form of a varying price level. What are the welfare consequences of these two effects? Given the overlapping generation structure of this economy we have to choose a welfare criterion to evaluate the effects of this policy. Let start by considering a 'golden rule' criterion and let look at the effect of this policy on the steady state utility of consumers born in odd and even periods.

Welfare analysis

Again it is convenient to consider first the benchmark economy with no bank-specific risk. In this case the consumption of odd-period consumers will be.

$$\begin{aligned}
\overline{\beta}\overline{c}_{11} &= \frac{\underline{p}}{\overline{p}}x + \frac{h}{\overline{p}} = \frac{m}{m+h}x + \frac{h}{m+h}(x+y) > x \\
\left(1-\overline{\beta}\right)\overline{c}_{22} &= \frac{\overline{p}}{\underline{p}}y - \frac{h}{\underline{p}} = \frac{m+h}{m}y - \frac{h}{m}(x+y) < y
\end{aligned}$$

the policy described has two effects on the steady state allocation: (1) it reallocates resources between odd and even period generations, and (2) it smoothes intertemporally the consumption path of even generation consumers. It is very well possible that this combination of effects generates a welfare improvement for both consumers. This does not contradict Proposition 1, because it disregard the transition from a fixed money supply to an active policy. Suppose that we begin with a policy of fixed money supply and we start implementing a policy of interest rate stabilization in an odd period, the effect of this policy will be to increase the price level from $\underline{p}$ to $\overline{p}$. This type of nominal instability hurts existing old consumers that have a fixed stock of money to spend on the goods market. This observation reflects the typical weakness of a 'golden rule' criterion. We can consider a simple modification of the golden rule criterion that includes the utility of existing old consumers. In particular, consider the parameters δ and θ defined in the proof of proposition 1. We can show that if the weight assigned to existing old consumers is $(1-\delta)$ and the weight assigned to odd and even period consumers are, respectively, θ and δ, then the optimal monetary policy is to set $h = 0$.

This benchmark case shows very well what is the basic mechanism by which central bank intervention affect the real interest rate in this economy. Late consumers in a liquidity abundant period t are indifferent between consumption at time t or at time $t+1$. On the other hand banks active in period $t+1$ would be happy to smooth their consumption profile by increasing the consumption of early consumers and reducing the consumption of late consumers. Monetary policy essentially induces the former group of late consumers to anticipate their consumption from time $t+1$ to time t (reducing $\underline{c}_{22}$ and increasing $\underline{c}_{21}$), freeing resources for early consumers in the next period.

When we introduce bank-specific risk monetary policy has a third effect, namely it changes the allocation of liquidity across banks active in the odd period. As we noticed in the previous section a reduction of the (real) interest rate in the odd period has the effect of tranferring resources from lending to borrowing banks, with a net positive effect in terms of welfare. Notice then that to study the liquidity reallocation effect we can adapt the analysis of the previous section on the welfare effects of an increase in the supply of reserves on the money market. The main difference is that the decision to hold reserves regarded solely the welfare of a given generation of consumers, while when the monetary authority intervenes, it necessarily affects the utility of the previous generation, by affecting the price level $\overline{p}$.

This monetary policy, though, cannot restore the economy to a Pareto optimal allocation. A Pareto optimal allocation requires a constant allocation across regions. Given that the all banks have the same endowment and the allocation of funds across banks is achieved through the operation of the money market, the only way in which c_{11}^k and c_{22}^k can be independent of k is when the real rate is 0. But the monetary policy we have considered implies that from odd to even periods there is a deflation, and thus cash has a positive return and by arbitrage it is impossible to have a zero real rate. The question remains open wether a more general monetary policy can drive to zero the real interest rate in odd periods, and possibly achieve a Pareto optimal allocation. As we have just noticed such a policy cannot entail a deflation (associated with a positive real rate of interest) and would necessarily entail inflation in odd periods. Nominal instability has real effects in this economy, by affecting the consumption of the existing old generation and by reallocating resources across odd-period and even-period consumers, so it is not clear that such a policy would be desirable.

Reserves requirements and interest rate stabilization

We have established that a monetary policy that injects funds in the money market in periods of scarce liquidity is effective in reducing (real and nominal) interest rate fluctuations, and that it may have positive welfare consequences by improving both intertemporal and cross sectional consumption smoothing for even-period born consumers. This is done, though, at the cost of an increased nominal variability and that affect the welfare of other generations. Suppose now, that the monetary authority tries to intervene solely on a given generation of odd-consumers. That is, suppose the central bank wants to concentrate on the constrained inefficiency due to the low level of reserves on the money market (Proposition 3), without affecting total nominal expenditure. The central bank can do that if it can tax active banks by withdrawing reserves in stage I and then lending these reserves back to them in stage II.In this way total expenditure remains fixed at m and the intervention has no consequences on other generations. This intervention, though, is completely ineffective if the tax in nominal term τ is smaller than $m - \alpha\overline{d}$. Here we are basically considering a case in which the central bank is not able to create liquidity and only tries to affect its allocation by changing its allocation over time. Such a situation is analogous to that studied, in a different context, by Caballero and Krishnamurthy (2001), and it comes as no surprise that we achieve a similar result of neutrality. As the central bank tries to save reserves on behalf of the banks the banks reduce their own reserves 1:1 correctly anticipating that the reserves will still be available on the money market. The only way to make this policy effective is to have $\tau > m - \alpha\overline{d}$, that is, to reduce the money stock in the hands of banks up to the point where they can no longer finance the original level of consumption of 'early leavers'. Only when reserves in the hands of banks have been driven down to zero, banks' behavior is constrained and the policy becomes effective. This, essentially amounts to a massive withdrawal of reserves immediately prior to a liquidity shortage followed by an equally massive injection of reserves in the money market. At this point, the central bank may well decide to attack the problem at its roots and instead of engeneering this massive open market operation it can impose a reserve requirement. A reserve requirement that imposes a level of reserves greater than the competitive level has the desired effect of reducing i on the open market, without causing nominal instability in the economy.

To sum up, we have considered two types of policy: standard open market operations and reserve requirements. With open market operations the central bank is actually able to *create* real liquidity for the current generation (the supply of real funds on the money market is $\frac{\underline{p}}{\bar{p}}x + \frac{h}{\bar{p}} > x$). It creates liquidity by increasing the price level and decreasing the value of money holdings of the old generation. This liquidity creation comes at the expense of nominal instability with side effects on the welfare of other generations. With a reserve requirement, instead, the central bank induces the banking system to *save* liquidity. This policy clearly has a more limited scope because the total level of liquidity available to the current generation is fixed, but it is able to address the ineffcent allocation of liquidity with no consequences for the price level.

Having established that stabilization of the interest rate on the open market and reserve requirements may concur to the same objective, an open question is what is the best combination of the two policies, and wethere they are complements or substitutes. One thing that is easy to show is that —without a reserve requirement— the level of reserve holdings w is lower in presence of a policy of interest rate stabilization. That is, when the central bank committs to supply reserves the reserves voluntarily held by private banks naturally decrease. At the same time, though, the open market policy improves the level of cross sectional insurance and thus also the second best level of reserves w^* is decreased. We do not expect to have unambiguous results on the effects of open market policy on the illiquidity of the banking system (as measured e.g. by $w^* - w$).

Another open question is wether other types of mechanisms may be more effective than rigid reserve requirements in dealing with the problem of inefficient reserves. Introducing some unobservable ex ante heterogenity in the regions it is easy to see that the second best level of reserves will depend on the individual distribution of the shocks β facing a given region. Therefore, a fixed reserve requirements valid for all banks may generate inefficiencies on this margin. On this regard notice that the use of the money market by a given bank may carry useful information on the distribution of its liqudity shocks. Recent practice in central banking seems to be oriented to a more intense use of information concerning a banks' activity on the money market. This type of model provides a rationale for this type of monitoring since it would allow to better target banks on the basis of thedistribution of their liquidity shocks.

5 Conclusion

We have developed a simple dynamic model of the interbank market that displays excessive interest rate fluctuations and insufficient holding of reserves by banks. By lending fiat money on the interbank market the central bank can stabilize the nominal and the real interest rate. In so doing it reallocates liquidity towards the banks with greater liquidity needs. This policy, though, generates nominal instability with potentially negative welfare consequences. The introduction of a reserve requirement attacks the same problem of insufficient liquidity without generating nominal instability. At the same time a a reseve requirement has a limited scope, because it cannot create additional liquidity on the money market. Therefore the legal reserve requirement (monetary control) and fiat money injections (in the spirit of the real-bills doctrine) can both be rationalized as policies oriented to support liquidity in the money market, with different advantages and costs. The effects of their joint use and their possible complementarities remain to be analyzed.

The 'pure' real-bills doctrine arrives at a very different conclusion: according to it a regime of free-banking would achieve the same allocation as interest rate stabilization *cum* reserve requirements. Rather than furnishing a useful companion to interest rate stabilization, reserve requirements are useless, at least from the point of view of monetary policy, and interest rate stabilization is actually needed to neutralize them. What accounts for this difference in results? The main difference between this paper approach and the 'pure' real-bills doctrine is that we focus on reserve money as an *input* in the provision of liquidity services by banks. From the point of view of the consumers bank deposits and hard currency are substitutes but from the point of view of banks reserves of outside money are an essential *input* in the supply of deposit money. While in Sargent and Wallace banks can create means of payment that are completely unbacked by hard currency reserves, in our paper hard currency backing is essential to the provision of credit services. To analyze in detail the consequences of this shift of focus this paper concentrates on the input role of reserves by keeping velocity and the money multiplier constant and equal to 1 at the aggregate level. In the model presented total expenditure grows always 1:1 with base money, and the quantity theory holds in a mechanical fashion. A single bank can expand credit beyond its money holdings by borrowing on the interbank market, but in the aggregate the amount of money available is fixed by the monetary authority. It is an interesting question for future research to understand how the mechanism of money multiplication affects the illiquidity problem of the banking system.

The contracts of liquidity provision offered by the banks in the model represent a mixture of deposits, contingent credit lines and other types of explicit or implicit committments. In the present formulation it is hard to map the different components of a bank's balance sheet into the unified contract of liquidity provision offered by the Diamond and Dybvig bank. Consumers make two deposits of cash at time t and $t+1$ and are allowed to withdraw different amounts of cash at period $t+1$ and $t+2$. This can be achieved with various mixes of: a variable rates of return on deposits and on credit lines. This degree of indeterminacy is not simply the result of theoretical parsimony, though. Banks have often used 'new financial products' to circumvent existing restrictions on credit expansion, that targeted specific pieces of their balance sheet (see e.g. the shift from the use of explicit credit lines to the use of commitments to subscribe commercial paper issues). Some countries have abandoned altogether the use of reserve requirements and have moved instead to a more intensive monitoring of day-by-day positions on the interbank market. This approach seems justified in the light of the analysis above, since the crucial source of problems is not the holding of reserves *per se*, but the fact that insufficient reserve holdings may lead to overborrowing ex post on the money market. Different banks with different exposure to bank-specific risk may want to keep very different reserve holdings, and a regulatory policy which is geared towards the use of money market information rather than towards the imposition of rigid reserve ratios should be able to better target the 'heavy users' of short term funds.

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6 Appendix

Lemma 5 *The economy in section 2 has a unique periodic equilibrium of period 2, with p constant and* $\underline{i} = 0 < \overline{i}$.

Proof. An equilibrium with $\underline{i} > 0$ is ruled out because total demand of funds on the money market in even periods would be smaller than $\underline{\beta}\,(x+y)$, and a fortiori smaller than x, which is only compatible with a zero interest rate. On the other hand, an equilibrium with $\overline{i} = 0$ is ruled out along the reasoning in the text. It remains to show that an equilibrium with $\underline{p} \neq \overline{p}$ is impossible. Such an equilibrium would be possible if total expenditure in even periods was smaller than m, which means that banks hold positive money balances in the middle age period. In that case we would have $\underline{p} < \overline{p}$ and a negative real interest rate. This implies that $\underline{c}_{22}^{k} = 0$ and total expenditure in period 1 equal to $\overline{p}x + \underline{p}y > x$, so equilibrium in the money market is impossible. ■

Proof of Proposition 1. Set the following parameters

$$\theta = \frac{u'(y/\left(1-\overline{\beta})\right)}{u'(x+y)}$$

$$\delta = \frac{u'(y/\left(1-\overline{\beta})\right)}{u'(x/\overline{\beta})} = \frac{1}{1+\overline{i}} < 1$$

Suppose that we start in period $t = 0$ (an analogous proof applies to the symmetric case). Consider the social welfare function which assignes weight

1 to consumers born at time -1, and the weight $\underline{\theta}\delta^m$ to consumers born in the even periods $2m$ and δ^{m+1} to consumers born in odd periods $2m+1$. Consider the problem of maximizing this social welfare function subject to feasibility in each period. The equilibrium allocation is an optimum of this problem, as can be checked by looking at the first order conditions.

Lemma 6 *The function $\phi(w)$ defined in the text is decreasing*

Proof. Consider the aggregate demand of funds on the money market $D(i,w) = \sum \left(\overline{\beta}^k - \alpha\right) \overline{c}_{11}^k$. Separability implies (1) that D is decreasing in i, and (2) that both c_{11} and c_{22} are normal goods, so that $\frac{\partial D}{\partial w} < \frac{1}{p}$. The function ϕ is defined by the condition $D(\phi(w), w/p) = w/p$, and the implicit function theorem gives the result desired. ■

Proof of Lemma 2. With a coefficient of relative risk aversion greater than 1 we have $Ru'(Rz) - u'(z) < 0$ for $R > 1$. This, together with the first order condition (4), implies that $(1+i)\, c_{11} > c_{22}$ for any shock β. The effect of $d\beta$ on bank's wealth is $-((1+i)\, c_{11} - c_{22}) d\beta < 0$, and since the marginal utility of income $u'(c_{11})$ is decreasing in total wealth we have the first part. The second part descends from separability. Suppose, by contradiction, that $\frac{d}{d\beta}\left((\beta - \alpha)c_{11}\right) < 0$ then we would have $\frac{d}{d\beta}\left((1-\beta)c_{22}\right) > 0$, these imply $\frac{d}{d\beta}c_{22} > 0 > \frac{d}{d\beta}c_{11}$, which contradicts (4).