Interruptible Physical Transmission Contracts for Congestion Management

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Abstract: This paper presents a novel congestion management protocol applicable for both pure-bilateral and hybrid market structure. The mechanism is based on an Interruptible Physical Transmission Contract, which guarantees physical access to the transmission network users and provides financial incentives for the bilateral contract holders to forfeit the physical access to the transmission network. The contract specifies a financial reimbursement payable to the insured if the system operator dispatch requires a curtailment of a bilateral contract involving an injection into and withdrawal of power at a specified set of nodes. The insurer is compensated in the form of an insurance premium for providing the service. The contract is structured such that the reimbursement payable to the insured party equals the actual loss incurred so that the insured party is "made whole" with the insurance payment. Similarly, when the insurer is the System Operator itself, it tries to dispatch the generators such that the aggregate insurance reimbursements payable to the insured parties is minimized. In this way, the transmission contract mechanism ensures that a near optimal curtailment policy coincides with the efficient dispatch in the system.

Keywords: Transmission Congestion Management, Financial Transmission Rights, Forwards, Options, Flowgate Rights, Strike Price, and Callable Forward

I. INTRODUCTION

It is generally agreed upon that an effective transmission congestion management system should provide adequate economic signals for an efficient use of the transmission grid in a simple manner [5]. In addition, the congestion management system should be able to accommodate long-term firm and non-firm bilateral contracts along with the real-time spot market. Different transmission congestion management protocols have been suggested so far based on these guidelines. The most notable among these include the Financial Transmission Rights or Transmission Congestion Contracts (TCC) [7], Flowgate Rights [4], and Usage based Physical Transmission Rights [2, 16].

In the TCC based approach, congestion management is performed through a bid-based centralized optimal power dispatch and the transmission rents are calculated *ex-post* as the nodal spot price differences. This approach yields the most optimal short-term dispatch solution. Combination of Location Based Marginal Price (LBMP) for spot congestion pricing and availability of long-term TCC based FTRs enable near optimal dispatch and bundled transmission and generation implementation of short-term reliability. However, only the holders of TCCs are risk neutral with respect to any unexpected equipment outages. This risk is automatically born by the Transmission Providers and/or consumers in case the sold TCCs are not simultaneously feasible in the actual system operation.

In the flowgate based approach, the System Operator defines and allocates a limited number of physical transmission rights that reflect the maximum power flow capacity across the transmission lines or groups of transmission lines [4]. The market participants who are interested in delivering electricity from one point in the electric power system to another are required to acquire a portfolio of such flowgate rights to back their energy transaction. An efficient secondary market is then developed for these flow-based rights, which could help the System Operator achieve optimal allocation of the transmission capacity. The approach has several advantages or features. For example, the flowgate rights can be assigned independent of the power flows and only the congested links require financial settlement. Moreover, the value of a flowgate right is never negative. However, although the decentralized nature of the flowgate rights based congestion mechanism is attractive, the approach suffers from several drawbacks. First, the number of flowgate rights that must be defined for allocation to interested parties may be significantly high [8]. Second, the transaction and information cost to traders may be very high [5]. Third, markets for these rights may be thinly traded and therefore efficient price discovery may be difficult. Finally, the traders may find it difficult to make informed decisions regarding their energy transactions since the flowgate rights are defined and allocated on a link basis whereas the traders are interested in point-to-point transactions only.

In the earlier approach presented in [2, 10] proposed a more general method for congestion management in which the Transmission Provider (TP) and system users are free to determine the terms of transmission provision and pricing iteratively over time. The transmission contracts presented in this paper could be easily implemented using this framework.

II. INTERRUPTIBLE PHYSICAL TRARNSMISSION CONTRACT

This paper proposes an alternative congestion management mechanism applicable for both pure-bilateral and hybrid market structure (where hour-ahead or day-ahead spot market coexists with the physical bilateral contracts with a varying degree of firmness). The mechanism is based on a novel Interruptible Physical Transmission Contract, which guarantees physical access for the market players to the transmission network and provides financial incentives for the bilateral contract holders to forfeit the physical access to the transmission network. The contract specifies a financial reimbursement payable to the insured if the system operator dispatch results in a curtailment of the bilateral contract involving an injection into and withdrawal of power at a specified set of nodes. The insurer (usually the SO and/or TP) is compensated in the form of an insurance premium for providing the service. The contract is structured such that the reimbursement payable to the insured party equals the actual loss incurred so that the insured party is "made whole" with the insurance payment. Similarly, when the insurer is the SO itself, it tries to dispatch the generators such that the aggregate insurance reimbursements payable to the insured parties is minimized. In this way, the transmission contract structure ensures that the optimal curtailment policy coincides with the efficient dispatch in the system.

Specifically, at the onset of any transaction, the bilateral transaction holder party (generator company and a load serving entity) initiates an interruptible physical transmission contract with a transmission provider, which specifies that whenever the bilateral transaction is curtailed, the transmission provider would pay the difference between the price prevailing in the load's zone[1] and the generator's self selected strike price[2] as compensation. The two parties (namely, transmission provider and bilateral transaction holder) agree on the maximum number of curtailments to be made during the life of the contract, the deductible applicable towards the insurance reimbursement and the contract price[3]. The hours during which the bilateral transaction could be curtailed is not specified at the origination of the contract. However, the number of times the transaction could be curtailed is specified in the contract. Moreover, the contract specifies that the load always receive replacement power at a price prevalent in its zone.

Thus, at the time of the origination of the contract, the two parties (bilateral transaction holder and Transmission Provider) agree to the following:

- Nodes at which generator and load would inject and withdraw power: i, j
- Time period during which the contract will be in force: T
- Maximum number of times the bilateral transaction could be curtailed: X
- Deductible amount: $C_{i,j}$
- Minimum and maximum amount of power to be curtailed at a time: q_{min}, q_{max}
- Total maximum quantity of power to be curtailed during the time period T: Q_{max}
- Price to be paid by the bilateral transaction holders to the TP for the transmission contract: $P_{i,j}$

It can be shown that the transmission provision contract just described is structured as a "Callable Forward" contract described in reference [6]. The bilateral transaction holder (generator and load in a pair) is "long" one forward contract and "short" one call-like option with a self-selected strike price. The call-like option is structured as a swing option due to the various flexibilities regarding the exercise rights included within the option. The discount that the bilateral transaction holder gets on the forward price is the swing option price at the time of contracting. The generator's self-selected strike price for the contract reveals the intrinsic value of the bilateral contract and the TP utilizes this information for efficient and economic allocation of the scarce transmission capacity in the real time. By subscribing to an interruptible contract with a self-selected strike price or deductible of $C_{i,j}$, the bilateral transaction holder reveals his reservation price for the bilateral contract. In the real time, the Transmission Provider utilizes this information for efficient and economic allocation of the scarce transmission capacity. The mechanism thus ensures that the optimal curtailment policy for the transmission provider accomplishes the economic dispatch of generators.

A. Methodology

In this subsection, we will develop a mathematical framework to calculate the price of the contract. For this purpose, let us first define the following variables:

T = Time period of the contract

t = Any instant during the life of the contract

k = Time step k

[1] Although the transmission contracts are explained with reference to a zonal electricity market structure, the method is equally applicable for a pure spot pricing based structure.

[2] Here the generator's self-selected strike price is similar to the deductible applicable in other insurance schemes.

[3] For the problem formulation, it is assumed that the System Operator (SO) is a for-profit entity and its revenue is regulated using the performance-based regulation. The specific form of the organization is however irrelevant for the discussion.

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N = Total number of time steps for time period T
S_k^i = Spot price at node i at time step k
S_k^j = Spot price at node j at time step k
$C_{i,j}$ = Deductible for the insurance contract
X = Maximum number of curtailments specified in the insurance contract
$I_{i,j}$ = Insurance payment to the bilateral transaction holder in the event of a curtailment
$I_{i,j} = S_j^k - C_{i,j}$
$F_{i,j}$ = Price of the forward contract component of the insurance contract
$J_{i,j}$ = Premium for the curtailment right (It is the discount a bilateral contract holder may get for subscribing for X number of curtailment during the life of the contract)
$P_{i,j}$ = Price of the Interruptible Physical Transmission Contract

With these notations, if the evolution of the prices at nodes i and j (S_k^i, S_k^j respectively) is given, the price of the contract can be calculated as follows.

Price of the contract, $P_{i,j}$ = Price of the forward component for the contract for the access to the transmission network – Discount which the bilateral transaction holder gets for allowing a maximum of X number of curtailments.

Therefore,

$$P_{i,j} = F_{i,j} - J_{i,j} \tag{1}$$

Where,

$$F_{i,j} = \hat{E}\{\sum_{k=1}^{N}(S_k^j - S_k^i)\} \tag{2}$$

To calculate $J_{i,j}$, please note that the overall profit of the transmission operator comprises of three components

1. Price of the insurance contract that the transmission provider receives at the beginning of the contract
2. Insurance payment to the bilateral transaction holder in the event of a curtailment
3. Transmission congestion rent that transmission provider collects from the alternate transmission network user in the event of a curtailment of the bilateral transaction holder

Thus, overall profit for the transmission provider is given by

$$\Pi = F_{i,j}(t,T) - J_{i,j}(t,T) + \{\sum_{k=1}^{N}(S_k^j - S_k^i) - \sum_{k=1}^{N} I_{i,j}\}u_k$$

$$= F_{i,j}(t,T) - J_{i,j}(t,T) + \sum_{k=1}^{N}\{(S_k^j - S_k^i) - (S_k^j - C_{i,j})\}u_k$$

$$\because I_{i,j} = S_k^j - C_{i,j}$$

$$= F_{i,j}(t,T) - J_{i,j}(t,T) + \sum_{k=1}^{N}(S_k^i - C_{i,j})u_k \tag{3}$$

Therefore, $J_{i,j}$ can be written as

$$J_{i,j} = \sum_{k=1}^{N} J_k \tag{4}$$

where,

$$J_k = \max_{u_k}\left\{\begin{array}{l} -\pi_k(S_k^i, u_k, w_k) \\ +\hat{E}(J_{k+1}(S_{k+1}^i, u_{k+1}, w_{k+1}))\end{array}\right\} \tag{5}$$

and

$$\pi_k = u_k(C_{i,j} - S_k^i) \tag{6}$$
$$S_{k+1}^i = f(S_k^i, w_k)$$
$$u_k = \{1(Curtail), 0(Wait)\}$$
$$\sum_{k=1}^{N} u_k \le X$$

For deriving Equation (6), it is assumed that the incremental effect of an additional transmission contract on the price processes within the system is negligible.

Bellman's principle of optimality for dynamic programming states that an optimal policy includes optimal sub-policies [3]. The stochastic optimization problem stated by Equations (4-6) solves for the optimal curtailment policy for the Transmission Provider. Therefore, the solution of the dynamic programming problem yields the optimal decision criteria for the Transmission Provider for each time step in the life of the transmission contract. More specifically, as implied by Equation (5), at each time step in the life of the Transmission Contract the Transmission provider is faced with the two choices:

1. Curtail the transmission provision to the bilateral transaction holder party and realize the payoff given by the difference of spot prices existing in the two zones less the insurance payment to the bilateral transaction party.
2. Wait until the next period.

The decision criterion for these choices define the individual optimal curtailment policy for a given time step and the aggregation of such individual sub-policies defines the overall curtailment policy for the Transmission Provider.

For the problem statement described above, we are considering the case of a single Interruptible Physical Transmission Contract. In real life, the Transmission Provider would have several such Interruptible Physical Transmission Contracts signed with many bilateral transaction holder parties. Therefore, the overall optimal curtailment policy would involve a combination of several optimal curtailment policies with the effect of each transmission contracts taken into account. Nonetheless, the basic principle for the decision criteria for the Transmission Provider would remain the same.

III. ALGORITHM FOR THE VALUATION OF THE TRANSMISSION CONTRACTS

In this subsection, we will develop a generalized algorithm, which will delineate the procedure to price the transmission right under the influence of different uncertainties for the scheme proposed in the paper.

In general, the pricing of the aforementioned "Callable Forward" contract employing the swing option could be done using the evolution of the prices at the generator and load nodes. Specifically, the forward contract price could be obtained by calculating the expected price of stochastic nodal price differential ($S_j - S_i$) existing between the nodes i and j. The call-type swing option could be priced using the stochastic price process of S_i at node i, the supply node as described in the earlier section. Unfortunately, the evolution of the price process is complicated for several reasons. First, electricity can't be stored and the load demand needs to be matched with the generation supply in real time on an instantaneous basis. Second, the power flow on a transmission network has to follow Kirchoff's laws. In the presence of transmission congestion, the resulting loop flows may be difficult to estimate due to various uncertainties that exist in a real-life power system, namely load variation, generation bid uncertainty and equipment outage uncertainty. One of the ways to circumvent these uncertainties and project the price processes is to use the Probabilistic Optimal Power Flow (POPF) as described in references [15, 17]. Specifically, assuming an appropriate load process, one could run POPF to estimate the evolution of the prices at different nodes. The resulting price processes could then be used to price the forward and swing option contracts.

A. Mean Reverting Price Process

The last subsection described a generalized algorithm for pricing an interruptible physical transmission contract under the influence of different price uncertainties. Such an elaborate algorithm may be appropriate for the Transmission Provider. Other market players may rely on the exogenous price processes to calculate the fair value of the contracts. Such price processes may range from the simple lognormal price process to more sophisticated supply-load dynamics based price process [13]. This section will utilize a one-factor mean reverting price process to illustrate the valuation of an interruptible physical transmission contract.

The swing options can be valued using a dynamic programming approach based on a discrete time approximation. For this purpose, the traditional binomial tree approach used for the lognormal price processes could be extended to trinomial trees in order to account for the mean reverting nature of electricity prices. Due to the multiplicity of the exercise rights, we need to construct a multi-layered trinomial model, also called as a "trinomial forest" [11]. Specifically, whereas the traditional American options can be exercised at any time during the life of the contract, they can be exercised only once. In contrast, the swing options have multiple exercises. Therefore, the optimal exercise policy includes several exercise opportunities, which could be incorporated using a multi-layer tree structure. This subsection describes the method for a single factor mean reverting price process. Similar procedure could be adopted for a two-factor model. The major difficulty to value any option for a mean reverting process is to build a trinomial tree. Therefore, we will first concentrate on tree-building procedure for a single-factor price process using the Hull-White approach [9].

The one-factor mean reverting price process assumes that the commodity price follows the stochastic process given by Equation (7).

$$dS = \alpha(\mu - S) + dz \tag{7}$$

In this equation,

S = Spot price

μ = Long-term equilibrium value of S

α = Mean reversion rate

σ = Volatility

dz = Weiner process

Defining $X = ln\ (S)$ and applying Ito's Lemma, we can describe the commodity price behavior by the Oornstein-Uhlenbeck stochastic process [12] as shown by Equations (8-9).

$$dX = \alpha(\overline{\mu} - \ln(S)) + dz \tag{8}$$

$$\overline{\mu} = \mu - \frac{\sigma^2}{2\alpha} \tag{9}$$

The first stage in building a tree for this process is to construct a tree for a variable S^* that is initially zero and follows the process

$$dS^* = -\alpha S^* dt + \sigma dz \tag{10}$$

This tree is symmetric about $S^* = 0$. The variable $S^*(t+\Delta t)-S^*$ is normally distributed. Our objective is to build a tree similar to the trinomial tree shown in Figure 2. To do this, we will need to identify which of the tree branch pattern shown in Figure 1 would apply for the tree drawn in Figure 2. It can be shown that if the branching pattern from any node (i,j) is as drawn in Figure 1(a), then the probability associated with the up branch, middle branch and down branch are given as p_u, p_m, p_d respectively which are given by the following equations.

$$p_u = \frac{1}{6} + \frac{\alpha^2 j^2 (\Delta t)^2 - \alpha j(\Delta t)}{2} \tag{11}$$

$$p_m = \frac{2}{3} - (\alpha j(\Delta t))^2$$

$$p_d = \frac{1}{6} + \frac{\alpha^2 j^2 (\Delta t)^2 + \alpha j(\Delta t)}{2}$$

Similarly, for the branch pattern shown in Figure 1(b), the probabilities are given as

$$p_u = \frac{1}{6} + \frac{\alpha^2 j^2 (\Delta t)^2 + \alpha j(\Delta t)}{2} \tag{12}$$

$$p_m = -\frac{1}{3} - (\alpha j(\Delta t))^2 - 2\alpha j(\Delta t)$$

$$p_d = \frac{1}{6} + \frac{\alpha^2 j^2 (\Delta t)^2 + 3\alpha j(\Delta t)}{2}$$

and for the one shown in Figure 1(c) they are,

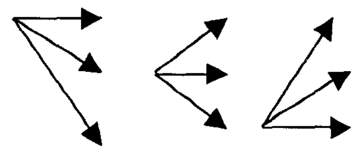
$$p_u = \frac{7}{6} + \frac{\alpha^2 j^2 (\Delta t)^2 - 3\alpha j(\Delta t)}{2} \tag{13}$$

$$p_m = -\frac{1}{3} - (\alpha j(\Delta t))^2 + 2\alpha j(\Delta t)$$

$$p_d = \frac{1}{6} + \frac{\alpha^2 j^2 (\Delta t)^2 - \alpha j(\Delta t)}{2}$$

Hull and White [9] have shown that the probabilities given by Equations (11-13) are always positive if j_{min} and j_{max} are set such that j_{max} is the smallest positive integer greater than $0.184/(\alpha\Delta t)$ and $j_{min} = -j_{max}$ where, j_{min} and j_{max} are the nodes at which the branching pattern changes from the one given by Figure 1(b) to Figure 1(c) and Figure 1(a) respectively. In the tree shown in Figure 2, the spacing between the spot rate ΔS is given as

$$\Delta S = \sigma\sqrt{3\Delta t} \tag{14}$$



(a) (b) (c)

Figure 1: Branch patterns for a tree for a mean reverting price process

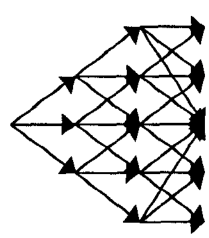


Figure 2: Trinomial Tree for S^* given by Equation (10)

Once the approximate tree for S^* is constructed the next stage is to displace the nodes on the S^* tree to so that the initial forward term structure is exactly matched. The branches shift in the new tree by an amount a_i for each time step i, but the probabilities are kept the same. In order to facilitate computations, let us define a new variable $Q_{i,j}$ as the present value of a security that pays \$1.00 if node (i,j) is reached and 0 otherwise. The variables a_is and $Q_{i,j}$s are calculated using forward induction. For instance, we assume that the $Q_{i,j}$s are calculated for time steps $i \leq m$. The next step is to calculate a_m such that the tree correctly prices the forward contract maturing at $(m+1)\Delta t$. The spot rate of electricity at node (m,j) is $a_m + j\Delta S$ so that the price of the forward contract maturing at $(m+1)\Delta t$ is given by

$$P_{m+1} = \sum_{-n_m}^{n_m} Q_{m,j} \exp(-(a_m + j\Delta S)\Delta t) \tag{15}$$

Solving Equation (15) we get

$$a_m = \frac{\ln(\sum_{-n_m}^{n_m} Q_{m,j} \exp(-j\Delta S\Delta t)) - \ln(P_{m+1})}{\Delta t} \tag{16}$$

Finally, $Q_{i,j}$ for the next time step can be calculated as

$$Q_{m+1,j} = \sum_k Q_{m,k} q(k,j) \exp(-(a_m + k\Delta S)\Delta t) \tag{17}$$

B. Swing Option Pricing

In order to value the swing option for the mean reverting process given by Equation (2), we first construct the discrete-time trinomial tree for the spot prices using the procedure described in the earlier section.

First, we will consider a ruthless version of the swing option wherein the quantity to be received at any exercise privilege is fixed [11]. Let us assume that the option provides two exercise privileges. In order to value the option, we can envisage 3 levels of trinomial trees (trinomial forest) one each for: no exercises left, one exercise left, two exercises left, and three exercises left.

1. At the bottom level, there are no exercises left, therefore the option price is zero.
2. At any other level, at any node, there are two options:
   - Exercise the option, and take delivery realizing the differential between the spot price and the strike price as the payoff.
   - Wait until the next period.

Accordingly, the option price at any node in any level is determined using the following equation:

$$J^k_{i,j} = \max\{S_{i,j} - C + e^{-rt}\hat{E}(J^{k+1}_{i+1}), e^{-rt}\hat{E}(J^k_{i+1})\} \tag{18}$$

where,

C = Strike Price

r = Risk-free interest rate

The above algorithm could be easily extended to price a more flexible swing option which allows any quantity to be received at any exercise within the minimum and maximum quantity limits (of course, in a discrete steps, for example, 10 exercises with the flexibility to receive any amount between 10 and 100 MW in discrete steps of 10 MW) [11]. In this case, the option pricing formula given for "ruthless" swing options is modified such that now there are multiple options to consider:

- Exercise the option, and take delivery of 1 unit realizing the differential between the spot price and the strike price as the payoff.

- Exercise the option, and take delivery of 2 units, 3 units, 4 units, and so on up to the maximum possible quantity specified in the contract.
- Wait until the next period.

Accordingly, the value of the option contract at any node in any level is determined by the following equation:

$$J^{k}_{i,j} = \max\{u(S_{i,j} - C) + e^{-rt}\hat{E}(J^{k+u+1}_{i+1}), e^{-rt}\hat{E}(J^{k}_{i+1})\} \quad (19)$$

where, $u = 1,2....(N-k)$ for any level k.

C. Pricing of the Forward Component

For the mean-reverting price process described by Equation (8), the forward contract price maturing t years from today can be given by [10]

$$\ln(F) = e^{-\alpha T}\ln(S_0) + (1 - e^{-\alpha T})\bar{\mu} + \frac{\sigma^2}{4\alpha}(1 - e^{-2\alpha T}) \quad (20)$$

Since the value of any forward contract maturing t years from today is given by

$$F(t) = \hat{E}(S(t)) \quad (21)$$

Equation (20) can be used in conjunction with Equation (2) to value the forward component of the transmission contract.

IV. EXAMPLE

In this section we will illustrate the valuation of the Interruptible Physical Transmission Contract for the California Power Exchange. We calibrated the historical zonal prices in the for the mean-reverting price process given by Equation (8). The parameters are given in the Table 1.

Table 1. Price Process Parameters for the Electricity Prices in California

Zone	Mean Reversion Rate	Annual Volatility
SP15	0.45721	75.58%
NP15	0 46811	57.36%
AZ2	0 47717	72.11%

Table 2. Interruptible Physical Transmission Contract price for AZ2-SP15 Power Delivery Contract

Example Data:
Generator Zone: AZ2
Load Zone: SP15
Time period of the contract: 1 year
Power delivery quantity: 1 MW

Curtailments	Forward Price	Option Price	Contract Price
5	$44959.97	$90 33	$44869 64
10	$44959 97	$176 67	$44783.30
20	$44959 97	$339 40	$44620.57
30	$44959 97	$492 00	$44467.97
50	$44959 97	$908.59	$44051 38

Table 2 describes the price of a transmission contract for delivery of power between zones AZ2 and SP15 for different number of curtailments.

Similarly, Table 3 gives the prices for the transmission contracts for different inter-zonal bilateral contracts.

Table 3: Interruptible Physical Transmission Contracts for Different Inter-zonal Bilateral Contracts

Generator Zone	Load Zone	Curtailments	Contract Price/MW
AZ2	SP15	20	$44618.57
AZ2	NP15	30	$44467.97
NP15	SP15	40	$2946.86

Although the forward and swing option components of the transmission contract are valued independently for the numerical example described above, the underlying mathematical framework is identical to the one described in Section 2 so far as the optimal decision criteria for curtailment for the TP is considered. In other words, the solution to the stochastic dynamic programming problem yields both the price for the transmission contract and the optimal transmission curtailment policy for the TP.

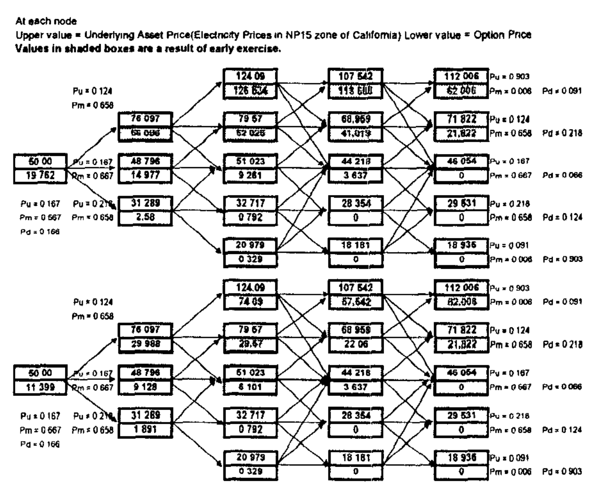


Figure 3: Trinomial Forest for swing option pricing

For illustration purposes, we will consider the valuation of a swing option for the transmission contract for the load residing in zone NP15. The example data and the procedure for valuation of the contract are explained in the Appendix. Figure 3 shows two layers of the trinomial tree for the contract for five time steps. The trees in the figure depict the likely spot price path for electricity in zone NP15 in California. The upper boxes in the tree show the spot price whereas the lower boxes show the corresponding option price at the given instant. The filled boxes for the option prices indicate the result of early exercises of the option contract. At each of these possible early exercises, the TP is likely to curtail the transmission contract and let other transmission users avail the transmission service since the spot price of electricity is relatively higher and would not justify waiting. Since there are only limited number of transmission curtailment privileges (two, in this case), the TP would need to weigh the odds of exercising the curtailment right at the given instant and waiting until the next possible exercise opportunity. Thus, at each of the exercise opportunity, the TP would need to compare the "option value" of waiting with the immediate payoff realizable with the exercise of one of the rights. To illustrate, at the third time step when only one curtailment right is left and the spot price of electricity happens to be $79.57/MW, the option value of waiting is $22.44/MW for the swing option whereas the immediate exercise of the option would fetch $29.57/MW. Therefore, the TP is likely to curtail the transaction at this particular instant. As described earlier, aggregation of such decision criteria defines the optimal curtailment policy over the life of the contract for the Transmission Provider.

V. CONCLUSION

A sound transmission congestion management protocol is essential for competitive electricity market. The interruptible physical transmission rights achieve market based congestion management for an efficient electricity market. Once implemented successfully, the interruptible transmission right based scheme yields long term optimal economic solution while facilitating optimal allocation of the scarce transmission capacity in real time thereby relieving network congestion.

VI. APPENDIX
SWING OPTIONS

Swing options have been in use in the energy markets for oil and natural gas. With the deregulation of electricity markets, they are also being used to hedge the risk in the electricity markets. The swing option gives the holder of the contract a right to repeatedly exercise an option up to a given number of times to receive any amount of energy within a specified limit. The exercise of the options has an implicit assumption for dependence on time. Thus, swing options gives the holder the flexibility to receive any amount of energy at any time within the constraints specified in the contract. These constraints relate to the quantity to be received at each exercise and the total amount to be received over the period of the contract. Following parameters and constraints define any swing option.

S_t = Spot price of the commodity at any time t
K = Strike price
r = Risk-free interest rate
T = Period of the contract
q = Quantity to be received at any exercise
Min_T, Max_T = Minimum and maximum quantity to be received during period T
Min_t, Max_t = Minimum and maximum quantity to be received at time t
N = Number of exercise rights
t_R = Refraction time period between consecutive exercises

Based on the above notations, swing options could be subjected to the following constraints:

$$Min_T \le \sum_{k=1}^{N} q \le Max_T \tag{22}$$
$$Min_t \le q \le Max_t$$
$$t_{i+1} - t_i \ge t_R$$

The first constraint defines the constraint related to the total quantity to be received over the time period T. The second constraint specifies the constraint regarding the amount that could be received at any exercise. Finally, the third constraint states that a minimum time period known as the refraction time necessary to be elapsed between two consecutive exercises.

Different variations of swing options could be constructed based on the option described above. The examples include:

- A pure timing option, in which the quantity to be received at each exercise is fixed. However, the option could be exercised multiple times at any time during the life of option.
- An option with a predetermined strike price, K, where the payoff at any exercise is determined by the difference between the spot price prevalent at time t and the pre-specified strike price K.
- An option where the strike price K is set equal to the spot or forward price observable at some future date.

ILLUSTRATION OF SWING OPTION PRICING

In this appendix we will illustrate pricing of a "ruthless" swing option for electricity prices in the NP15 zone of California.

<u>Example Data:</u>

Load Zone: NP15
Volatility, $\sigma = 0.5736$
Mean reversion rate, $\alpha = 0.46811$
Initial spot price, S_0 = \$50.00/MW
Strike price, K = \$50.00/MW
Life of the contract, $T = 1.0$ year
Number of rights, $X = 2$
Number of time steps, $N = 5$

The "trinomial forest" for the option is as shown in Figure 3. In this Figure, at each node, the values in the upper box represent the underlying electricity price and the values in the lower box represent the option price. The filled lower boxes represent the results of early exercises. For example, the second element in the third column of the bottom tree represents an early exercise with a payoff of $29.57 = ($79.57-$50.00). The bottom level of the "forest" represents the tree when one of the two rights has been exercised. Similarly, the upper level represents the trinomial tree when there are two rights left to be exercised. To illustrate the procedure, the element 29.57 in bottom level tree (2nd row, 3rd Column) is obtained as

$29.57 = max\{(79.57\text{-}50.0)+0,\ (57.542P_u + 22.06P_m + 3.637P_d)\}$

where, $P_u = 0.124$, $P_m = 0.658$, and $P_d = 0.218$.

Similarly, the element 113.688 (1st row, 4th column) in the top tree is obtained as

$113.688 = max\{(107.542\text{-}50.00)+(62.006P_u+ 21.822P_m + 0.0P_d),\ (62.006P_u+ 21.822P_m + 0.0P_d)\}$

where $P_u = 0.903$, $P_m = 0.006$, and $P_d = 0.091$

As the number of time steps N increases the option price converges to its exact value.

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