Localized Response Performance of the Decoupled $Q-V$ Network

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Abstract —**In this paper the assumption of a localized voltage response due to reactive power disturbances is analyzed. Conditions on transmission line parameters, given its normal operating point, are stated which explicitly define directions of voltage changes so that the line is subject to the localized voltage response. We show that it is not possible to give an exclusive answer to the question of the localized response (tier-wise) in the decoupled $Q-V$ network. The answer is network and operating point dependent. The operating regions in which this property is satisfied even under large changes in reactive power injections are derived on the $S-E$ graph based decoupled $Q-V$ network. We first define a no-gain operating mode of this network and then claim that the power no-gain operating mode always implies a localized voltage response.**

These results cannot be used to demonstrate a voltage gain. We develop algebraic type statements to show that a system may have response which is system wide. More definite answers on the localized response are established for the echelon structure of a given network.

I. Introduction

REACTIVE POWER problems arise in power networks under a variety of conditions: for lightly loaded systems too much reactive power may be injected into the network by the shunt elements with two important consequences: 1) the bus voltages at voltage uncontrolled busses become overly high; or 2) the extra reactive power has to be absorbed by the system generators causing potentially damaging under-excitations. Alternatively under heavy load conditions there may be insufficient injected reactive power causing the voltages to drop. This situation may worsen with the loss of a generator, but the loss of a line has also an effect on the balance [13].

In this work we study theoretical aspects of the reactive power imbalance problem in steady state under large load changes. It is known that the transmission of reactive power through a power network poses more serious problems than its generation. This is partly attributable to the large reactive power losses relative to the transmitted reactive power, the ratio of the loss and transmitted reactive power being much larger than in the case of active power flows. A network formulation of the reactive power transmission is missing even for the linearized version of the reactive power network, although some work has been done following a series of blackouts caused by reactive power transmission problems [1], [2]. Most of the work done for the nonlinear $Q-V$ network uses the off-line load flow analysis or nonlinear programming techniques to study selected sets of critical disturbances [14], [15]. There is still not much work done for the nonlinear $Q-V$ network which would provide an adequate model for studying analytical properties of this network without having to use time consuming tools like [14], [15]. Our work is intended to develop specific properties of the $Q-V$ network which would make the analysis easier. We comment on the relation of some recent results in this direction [3], [4] to our work.

The main problem that we address here is the justification of the assumption of localized response to changes in reactive power injection. Here, by a localized response, we mean a similar phenomenon to the one introduced in [6] for the active power-phase angle $P-\theta$ network: if a generator or load outage occurs which causes a change in the bus voltage magnitudes, then the change is largest at the location where the fault has occurred and this change propagates through the system tierwise [7]. Here, by tier N we mean all buses directly connected to the buses belonging to tier $(N-1)$, $N=1, 2, \cdots, k$, where $N=1$ indicates the location where the fault has occurred. In order to define operating regions in which this property is satisfied even under large changes in reactive power injections, we propose to use the $S-E$ graph based decoupled nonlinear $Q-V$ network and apply to it some results from the nonlinear electric network theory [8], [9]. We propose this network as the only known way to treat the reactive power–voltage problem as a nonlinear network problem when the reactive power losses are included. An indirect approach to this is taken in a sense that we define first a no-gain operating mode [10] of the nonlinear $Q-V$ network. This result is interesting enough by itself for applications to security monitoring. It states that if there is a disturbance in a reactive power injection $|\Delta Q_i|$ due to a generator or load outage, then the change in the reactive power flow along any transmission line in the system is smaller than or equal to $|\Delta Q_i|$. This means that if one knows security constraints and the normal operating point, then the changes in power flows due to disturbance $|\Delta Q_i|$ can be estimated for all transmission lines and the security

Manuscript received August 11, 1983; revised August 12, 1985. This work was supported in part by the National Science Foundation under Grant ECS-8214143 and grant ECS-8214262.

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IEEE Log Number 8406681.

0098-4094/86/0300-0316$01.00 ©1986 IEEE

limits checked without doing time consuming load flow calculations.

Next we establish relations between the no-gain operating mode and the localized response performance in the $Q-V$ network. To discuss the systemwide response we develop algebraic results for the nonlinear $Q-V$ problem where we show that the system is not always subject to a localized response.

In conclusion, we show that it is not possible to give an exclusive answer to the question of the localized response in the reactive power–voltage problem. The answer is network and operating point dependent. We state mathematical conditions under which a localized response is true. As a consequence of this, a straightforward usage of this phenomenon is not possible in monitoring or controlling the power network. This established result is negative in nature; i.e., it claims that a localized $Q-V$ response (tier wise) does not generally exist in typical power networks. Therefore, to state more specific conditions on localized response for a given network a concept of an echelon is introduced. Unlike the tier concept, the echelon is defined *independently* of disturbance location: echelon 1 of a given power network consists of a set of all PV buses; echelon 2 is formed by a set of all PQ buses directly connected to echelon 1 (and not belonging to it); echelon 3 consists of a set of all PQ buses directly connected to the buses in echelon 2 (not belonging to it), etc.

II. The S–E Graph Based Decoupled $Q-V$ Network

The background on the $S-E$ graph is given in [8], [9]. Here we consider only the reactive power flows Q and assume that $Q=Q(V)$ is a function of the bus voltage magnitudes V with the phase angles θ (known) constant. *Note:* Our work is based on the assumption that $Q-V$, $P-\theta$ decoupling is justified. In the large disturbance problem θ may be very coupled to Q (especially in the voltage collapse problem) and the developed results under the decoupling assumption may not always be valid. An estimate of the largest error in a given network due to decoupling error can be derived using results of Kaye and Wu [16]. Similar results have been obtained recently which include the effect of shunts on the network performance and estimate of the decoupling error [17]. It should be noted, though, that since most of the results in the work on the localized response are expressed in terms of inequalities rather than equalities, it can be shown that similar modified results are rather easy to obtain for a physically meaningful range of phase angles $|\theta_{ij}|<\pi/2$ with the coupling taken into account. We illustrate the modification process in Section IV, where the modified results take into account coupling with the phase angles. In this light, the decoupling assumption is justified for the techniques developed in our work, or it can be corrected for in a relatively straightforward way.

In the $S-E$ graph based $Q-V$ network loads are *independent* reactive power sources, generators are *independent* voltage (magnitude) sources (including slack bus) and each transmission element has a well-defined $Q \simeq V$ constitutive relation as

$$Q_{ij}^{T}=\mathrm{Im}\left(S_{ij}^{T}\right)=f^{T}(V) \tag{1}$$

$$Q_{ij}^{L}=\mathrm{Im}\left(S_{ij}^{L}\right)=f^{L}(V). \tag{2}$$

In particular, for a transmission line ij, a complete model with shunt capacitances included is

$$Q_{ij}^{T}(V_i,V_j)=V_iV_jB\cos\theta_{ij}-V_j^2B, \qquad B>0 \tag{3}$$

$$Q_{ij}^{L}(V_i,V_j)=\left(V_i^2+V_j^2-2V_iV_j\cos\theta_{ij}\right)B, \qquad B>0 \tag{4}$$

$$Q_{i0}=-V_i^2B_{is}, \qquad B_{is}>0 \tag{5}$$

$$Q_{j0}=-V_j^2B_{is}, \qquad B_{is}>0. \tag{6}$$

Note that his model is defined so that reactive power flows satisfy the flow conservation law [8]. The primitive elements after interconnecting as defined by the one-line-diagram of a power system form the decoupled $Q-V$ network. We call this network a $Q-V$ network to emphasize that it is analogous to the $I-E$ network, where the reactive power flows satisfy conservation of flow law instead of currents I in an electrical network. Here the primitive elements of transmission lines belong to the three terminal type elements for which some interesting network theory properties are introduced in [10]. We use this work to establish properties of the nonlinear $Q-V$ power network.

III. No Gain Operating Mode of the $Q-V$ Network and Its Localized Response

Most of the results in this section on the no-gain operating mode and localized response for the decoupled reactive power–voltage magnitude $Q-V$ network are analogous to the results derived in [12] for the active power-phase angle $P-\theta$ decoupled network and we shall therefore omit many of the details. The interested reader can easily translate the results from [12] to present context.

By a localized response in the $Q-V$ network, we mean the following: if a generator or load outage occurs which causes a change in the bus voltage magnitudes, then the change is the largest at the location where the fault has occurred and this change propagates through the system tierwise [6]. Here by tier N, we mean all buses directly connected to the buses belonging to tier $(N-1)$, $N=1,2,\cdots$, where $N=1$ indicates the location where the fault has occurred.

Definition 1: The $Q-V$ (sub)network is said to possess the no-gain reactive power (voltage magnitude) property in some operating region $S\in\Omega$ if the magnitudes $|\Delta Q_{ij}^{T}|$, $|\Delta Q_{ij}^{L}|$ (voltage magnitudes $|\Delta V_i$, $\Delta V_j|$) of the changes in transmitted and loss reactive power for each pair of nodes i and j (voltage magnitudes of buses i and j) are less than or equal to the sum $\Sigma|\Delta Q_s|$ of the magnitudes of the changes in reactive power through the independent power sources, (sum $\Sigma|\Delta V_s|$ of the magnitudes of changes in voltage magnitudes across independent voltage sources) provided $\Delta V_i \in \Omega$, $i = 1, 2, \cdots, N$.

Definition 2: A three-terminal $Q-V$ element ϵ is said to be a no-gain element if for any $Q-V$ network possessing the no-gain property, if ϵ is connected to the network in any manner there is a nonempty set $S \subset R^N$ such that the new network possesses the no-gain property in S.

We then have the following:

Theorem 1: The transmission line $i-j$ is a no-gain element in $S \subset R^2$ if

$$\Delta V_i \Delta Q_i > 0 \qquad \text{if } \Delta V_j / \Delta V_i < 1 \quad (7)$$

$$\Delta V_j \Delta Q_j > 0 \qquad \text{if } \Delta V_i / \Delta V_j < 1 \quad (8)$$

$$(\Delta V_i + \Delta V_j)(\Delta Q_i + \Delta Q_j) > 0 \qquad \text{if } \Delta V_i \Delta V_j > 0. \quad (9)$$

The proof of this theorem is identical to the proof of theorem in [10].

Note: Not all points in the region $(\Delta V_i, \Delta V_j)$ defined by Theorem 1 satisfy the load flow equations. If disturbance occurs which causes voltage changes ΔV_i, ΔV_j on a given transmission line to fall in the no-gain area then the line will possess the no-gain property at a new operating point $(V_i + \Delta V_i,\ V_j + \Delta V_j)$. No claims are made on the sets of disturbances which make this possible on a given power network. Some further research is needed to interpret the no-gain operating regions of separate transmission lines on a connected power network.

One needs to stress the effect of the parameters of the transmission line and nominal operating point on the size and shape of the no-gain operating region. To the author's knowledge, this is the first time that a nonlinear analysis of the $Q-V$ transmission problem has been done in the literature.

It is possible to check conditions (7)–(9) and find operating regions for which such transmission line is in the no-gain operating mode. However, for practical applications, one would like to have an open set with a certain r around the normal operating point and then claim that for any disturbance which causes change in voltages ΔV_i and ΔV_j such that, for example

$$\Delta V_i^2 + \Delta V_j^2 \leq r^2 \quad (10)$$

the transmission line will be in the no-gain operating mode. Simulations show that what is more likely to happen for small transmission lines is that only a certain subset of R^2 around the operating point V_i^0, V_j^0 satisfies the no-gain operating mode conditions. Only transmission lines which have large $|V_i^0 - V_j^0|$ and/or large B/B_s and/or large θ_{ij}^0 will be in the no-gain operating mode for any direction of change ΔV_i, ΔV_j. Chua has introduced a concept of *locally* no-gain three terminal elements in [11] which if used on the $Q-V$ network states conditions for the transmission line to be locally in the no-gain operating mode. It can be shown that these conditions are very restrictive which implies that many transmission lines are subject to gains. We have checked these conditions for the 14 bus AEP system and shown that none of the transmission lines are locally no-gain. Only three lines on the 118 IEEE system are locally in the no-gain mode. Still conditions (7)–(9) show a larger region around the operating point where lines behave as no-gain elements.

The idea of a localized response in the $Q-V$ power network is used in practice for computational savings, although not always justified. Some similar results are known to power engineers: If we consider a Q-disturbance at a bus surrounded by voltage controlled buses, in the first tier no voltage change will take place. In subsequent tiers with some voltage *uncontrolled* buses the voltage will however change. In addition, power system engineers have a fairly good feeling from simulation studies of the nonlocalized effect of Q-injection disturbances in general. For example, the voltage at certain PQ buses is often regulated by injecting reactive power at nearby PQ buses without modifying the latter's voltages by much. It is known that this is possible if at the buses where Q is being injected there exists a relatively large shunt capacitance compared to the series reactance leading to the regulated bus.

Again, analogous to the case in [12] we have

Theorem 2: The power no-gain operating mode in a power system always implies the localized voltage response: the smallest change in tier I is larger or equal to the largest change in tier II, etc.

$$|\Delta V_i| \geq \|\Delta V_I\|\infty \geq \|\Delta V_{II}\|\infty \geq \cdots. \quad (11)$$

Using this approach, one may conclude that most of the transmission lines of a power network possess the no-gain property only in a subset of R^2. Furthermore, transmission lines which have large B and/or θ_{ij}^0 at the normal operating point and small B_s will have larger power no-gain property region in a given power network.

IV. On a Nonlocalized Response in the $Q-V$ Network

Although results in the previous section explicitly define regions where a transmission line has a localized response, these results are not applicable to the not no-gain claims; i.e., it is not true that if the not no-gain transmission lines are connected together, the network would be a gain-network. As a consequence, we cannot say anything about a systemwide response (as opposed to localized) based on the no-gain theory results.

Therefore, we develop algebraic type statements to prove that a system may have response which is systemwide. These results do not state explicitly the operating regions where the systemwide response could occur. To establish the nonlocalized response of the $Q-V$ network and to demonstrate the relationship between the shunt elements and the nonlocalized response it is necessary to consider the reactive power balance equations at each bus. The reactive power injection at bus i is written

$$Q_i = -V_i \sum_{k \in K_i} V_k c_{ik} \quad (12)$$

where K_i are the buses connected directly to bus i (including i) and

$$c_{ik} = c_{ki} = B_{ik} \cos \theta_{ik}, \qquad i \neq k \quad (13)$$

and

$$c_{ii}=B_{ii}=-B_{is}-\sum_{\substack{k\in K_i\\ i\neq k}} B_{ik}. \tag{14}$$

To clarify the sign convention in (12), notice that for inductive transmission lines $B_{ik}<0$ and capacitive shunts $B_{is}>0$ where B_{is} is the shunt connection at bus i. Since the V_i are voltage magnitudes, write

$$V_i=e^{xi} \tag{15}$$

to form

$$Q_i=\sum_{k\in K_i} e^{(x_i+x_k)}c_{ik} \tag{16}$$

or

$$Q_i=-\sum_{k\in K_i} f^{(x_i+x_k)}c_{ik}. \tag{17}$$

Since we are concerned with perturbations of the solutions, we form

$$\Delta Q_i=-\sum_{k\in K_i}\left[f(x_i+x_k+\Delta x_i+\Delta x_k)-f(x_i+x_k)\right]c_{ik} \tag{18}$$

or

$$\Delta Q_i=-\sum_{k\in K_i} h_{ik}(\Delta x_i+\Delta x_k)c_{ik} \tag{19}$$

where

$$h_{ik}=e^{(x_i+x_k)}\left(e^{(\Delta x_i+\Delta x_k)}-1\right). \tag{20}$$

It can be seen that $h_{ik}(\Delta x_i+\Delta x_k)$ is monotone increasing and restricted to the first and third quadrants in $(\Delta x_i+\Delta x_k)$. We create a linear system of equations of the form

$$\Delta Q_i=-\sum_{k\in K_i}(\Delta x_i+\Delta x_k)c_{ik}g_{ik} \tag{21}$$

where $g_{ik}=g_{ki}$. If we examine solutions of (18) for which $|\Delta x_j|<r\ j=1,2,\cdots,N$ then it is only necessary to consider g_{ik} in (21) which are bounded by

$$V_iV_k\frac{(e^{-2r}-1)}{-2r}\leqslant g_{ik}\leqslant V_iV_k\frac{(e^{2r}-1)}{2r}. \tag{22}$$

Further, if we are interested only in problems of voltage reduction $-r\leqslant\Delta x_j\leqslant 0\ j=1,2,\cdots,N$ it is only necessary to consider g_{ik} which are bounded by

$$V_iV_k\frac{(e^{-2r}-1)}{-2r}<g_{ik}\leqslant V_iV_k. \tag{23}$$

Every solution to (18) with bounded Δx_j then corresponds to a solution of (21) with some set of coefficients g_{ik} bounded by inequalities (22) or (23). To demonstrate the nonlocalized response of (18), it is necessary then to demonstrate nonlocalized response for the linear system (21) for all g_{ik} within the appropriate bounds. We will show that the set of equations (21) have a network interpretation and that there are disturbances ΔQ_i that produce a nonlocalized response. To put the linear system in more recognizable form, rewrite (21) as

$$-\Delta Q=H\Delta x \tag{24}$$

where

$$h_{ik}=c_{ik}g_{ik}=g_{ik}B_{ik}\cos\theta_{ik} \tag{25}$$

and

$$h_{ii}=\sum_{\substack{k\in K_i\\ k\neq i}} c_{ik}g_{ik}+2c_{ii}g_{ii}. \tag{26}$$

The conventions involved in (12) produce nonpositive off-diagonal entries in the $\boldsymbol{B}$ matrix for the power system. If we associate $-\Delta Q$ with a set of currents and Δx with a set of node voltages, then the $\boldsymbol{H}$ matrix can be thought of as a conductance matrix for a resistive network with the same topology as the power system. Nodes i and k in the resistive network are connected with positive conductance $-g_{ik}B_{ik}\cos\theta_{ik}$. The conductance to ground at node i is given by

$$h_{is}=2g_{ii}c_{ii}+2\sum_{\substack{k\in K_i\\ k\neq i}} c_{ik}g_{ik} \tag{27}$$

$$h_{is}=2g_{ii}\left[-B_{is}-\sum_{\substack{k\in K_i\\ k\neq i}} B_{ik}\right]+2\sum_{\substack{k\in K_i\\ k\neq i}} g_{ik}B_{ik}\cos\theta_{ik}. \tag{28}$$

To verify that the conductances h_{is} are typically negative consider perturbations small enough so that

$$g_{ik}\approx V_iV_k. \tag{29}$$

In this case

$$h_{is}\approx -2Q_i. \tag{30}$$

The typical load bus (PQ bus) in the power system has $Q_i>0$ so that for small disturbances the equivalent resistive network is composed of positive resistors between nodes and negative resistors to ground at the nodes corresponding to load buses. The dc load flow assumptions of small angles θ_{ik} and voltage magnitudes near unity give, for small perturbations

$$h_{is}\approx -2B_{is}. \tag{31}$$

Again, the typical PQ bus has a capacitive connection to ground from the transmission line models and possible static capacitors which with our sign convention is a negative value of h_{is}.

Estimates of the size of the perturbations r, for which the shunt connections remain negative can also be obtained. If we assume

$$\left[-B_{is}-\sum_{\substack{k\in K_i\\ k\neq i}} B_{ik}\right]>0 \tag{32}$$

which is typical at PQ buses then h_{is} is maximized if g_{ii} takes on the maximum value possible while each g_{ik} is the minimum. For example, for the voltage reduction case

$$h_{is} \leqslant Q_{is} + \frac{(1-e^{-2r})}{2r} Q_{i2} \tag{33}$$

where

$$Q_{is} = 2V_i^2 \left[B_{is} - \sum_{\substack{k \in K_i \\ k \neq i}} B_{ik} \right] \tag{34}$$

and

$$Q_{i2} = 2 \sum_{\substack{k \in K_i \\ k \neq i}} V_i V_k B_{ik} \cos \theta_{ik}. \tag{35}$$

Since Q_{is} is available from the diagonal entry of the Jacobian of the load flow solution and

$$Q_i = -\frac{(Q_{is} + Q_{i2})}{2} \tag{36}$$

the range of r for which (33) is negative can be computed directly. For the AEP 14 bus system the bounds on r at the load buses were computed. The corresponding voltage bounds ranged from 22 percent to 65 percent. In other words, for all acceptable perturbations of the 14 bus system the equivalent resistive network has negative shunt connections to ground at the nodes corresponding to PQ buses.

With the preceding as justification we write

$$H = \overline{B} - \overline{C} \tag{37}$$

where $\overline{B}$ is an admittance like matrix and $\overline{C}$ is a diagonal matrix representing the negative conductances to ground from (28). From (25)

$$\overline{B}_{ik} = g_{ik} B_{ik} \cos \theta_{ik}$$

where the g_{ik} satisfy (22) or (23).

To examine the structure of disturbance propagation, we define an *echelon* structure for the network. Let echelon 1 correspond to the PV buses, echelon 2 to the PQ buses (not in echelon 1) directly connected to echelon $1, \cdots$ echelon i to PQ buses (not in previous echelons) directly connected to echelon $(i-1)$, etc. Three or four echelons seem typical of transmission systems we have examined. It should be noted that echelons are different than organizations of the network that are disturbance related like tiers [7] or disturbances areas [19]. We can write (24) as

$$\begin{bmatrix} \overline{B}_{11} - \overline{C}_{11} & \overline{B}_{12} & 0 & 0 \\ \overline{B}_{21} & \overline{B}_{22} - \overline{C}_{22} & \overline{B}_{23} & 0 \\ 0 & \overline{B}_{32} & \overline{B}_{33} - \overline{C}_{33} & \overline{B}_{34} \\ 0 & 0 & \overline{B}_{43} & \overline{B}_{44} - \overline{C}_{44} \end{bmatrix} \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \\ \Delta x_3 \\ \Delta x_4 \end{bmatrix} = - \begin{bmatrix} \Delta Q_1 \\ \Delta Q_2 \\ \Delta Q_3 \\ \Delta Q_4 \end{bmatrix} \tag{38}$$

or

$$H \Delta x = - \Delta Q$$

where $\overline{B}_{ii} > 0$, $\overline{B}_{ij} \leqslant 0$, and $\overline{C}_{ii}$ is a diagonal matrix with $C_{ii} > 0$ representing negative connection to ground.

V. Nonnegative Matrices

Such an H matrix is called an M matrix if it can be written as

$$H = sI - A \qquad \text{where } A \geqslant 0$$

and $\rho(A) \leqslant s$, where $\rho(A)$ is the spectral radius of A and $A \geqslant 0$ means that every element of A is nonnegative. If $s > \rho(A)$, then H is a nonsingular M matrix. There are a large number of equivalent conditions to the statement "H is a nonsingular M matrix" [20]. The following are equivalent and are used in the sequel:

1) H is a nonsingular M matrix;
2) $H^{-1} > 0$;
3) all the principle minors of H are positive;
4) There exists a positive diagonal matrix, D such that HD is strictly dominant.

An immediate consequence of 4) is that if H is a nonsingular M matrix, then every principle submatrix of H has a nonnegative inverse. Since all off-diagonal entries are nonpositive

$$(\overline{B}_{ii} - \overline{C}_{ii}) D_i, \qquad i = 2, 3, 4$$

is strictly dominant and $(\overline{B}_{ii} - \overline{C}_{ii})^{-1} \geqslant 0$.

In the sequel it will be assumed that H is a nonsingular M matrix. Note H itself is not diagonally dominant due to the shunt connections, but has both positive and negative row sums. The positive row sums correspond to the connections to the swing bus or shunt reactors in the full H matrix of (38). The assumption that H is a nonsingular M matrix does not preclude shunt connections but limits their size. To see that the limit is a reasonable one, consider a H with some negative shunts but which is a nonsingular M matrix and imagine increasing one shunt, i.e.,

$$\left(\overline{B} - \bar{c}_{ii} e_i e_i^T \right)^{-1} = X + \frac{\bar{c}_{ii} x_i x_i^T}{1 - \bar{c}_{ii} x_{ii}} \tag{39}$$

where $H^{-1} = X \geqslant 0$ and x_i is the ith column of X, e_i is a vector with a one in the ith position and zeros elsewhere. The limiting value of $\bar{c}_{ii}$ is seen to be $1/x_{ii}$. The inverse remains positive until it blows up. For $\bar{c}_{ii} = \epsilon + 1/x_{ii}$, however, the inverse is entirely negative. Critical values of shunt capacitance for example systems are found to be unreasonably large.

In general, we will assume that the PV buses regulate correctly, i.e., that $\Delta x_1 = 0$. Before proceeding, however, suppose that $\Delta x_1 = \alpha e_i$, i.e., a small perturbation in one PV bus voltage while the other PV buses maintain voltage. The remaining voltages must satisfy

$$\begin{bmatrix} (\bar{B}_{22}-\bar{C}_{22}) & \bar{B}_{23} & 0 \\ \bar{B}_{32} & (\bar{B}_{33}-\bar{C}_{33}) & \bar{B}_{34} \\ 0 & \bar{B}_{43} & (\bar{B}_{44}-\bar{C}_{44}) \end{bmatrix} \begin{bmatrix} \Delta x_2 \\ \Delta x_3 \\ \Delta x_4 \end{bmatrix} = \begin{bmatrix} -\alpha \bar{B}_{21} e_i \\ 0 \\ 0 \end{bmatrix} \quad (40)$$

or

$$\hat{H}_{22} \Delta x = -\alpha \Delta Q.$$

If there are no shunt reactors in the second echelon, the row sums in the matrix $\hat{H}_{22}$ of (40) are

$$(\hat{H}_{22} 1)^T = \left[(-\bar{B}_{21} 1 - \bar{c}_2)^T : -\bar{c}_3^T : +\bar{c}_4^T \right]$$

where 1 is a vector of 1's, and $\bar{c}_i = \bar{C}_{ii} 1$. Reactors to ground would only increase the first part of the row sum. Since $\bar{B}_{21} \leqslant 0$ and $\hat{H}_{22}$ is a principle submatrix of H, it can be seen that the assumption that H is a nonsingular M matrix implies that the lower echelon voltages track the PV buses in the sense that if a PV bus voltage increases, then the lower echelon voltages tend to increase and visa versa. Again, if H fails to have a positive inverse, unusual system behavior can be expected.

The matrix H_{22} is significant in investigating other disturbance propagation. If $\Delta x_1 = 0$, then arbitrary perturbations must satisfy where $\hat{H}_{22}$ has positive row sums in echelon 2 and negative row sums in echelons 3 and 4. We now state results which are independent of the size of the shunts as long as H has a positive inverse and has the assumed row sums.

VI. Uniform Perturbations

Consider a nonsingular M matrix partitioned as follows

$$\begin{bmatrix} M_{aa} & M_{ab} & 0 \\ M_{ba} & M_{bb} & M_{bc} \\ 0 & M_{cb} & M_{cc} \end{bmatrix} \begin{bmatrix} \Delta x_a \\ \Delta x_b \\ \Delta x_c \end{bmatrix} = \begin{bmatrix} 0 \\ -\Delta Q_b \\ 0 \end{bmatrix} \quad (41)$$

where

$$M_{aa}1 + M_{ab}1 = k \geqslant 0$$
$$M_{cb}1 + M_{cc}1 = -c; \qquad c \geqslant 0$$

and let ΔQ_b be chosen so that $\Delta x_b = \alpha 1$ a "uniform" perturbation. Then it can be verified that the solution to (41) is given by

$$\Delta x_a = \alpha\left(1 - M_{aa}^{-1} k\right)$$
$$\Delta x_c = \alpha\left(1 + M_{cc}^{-1} c\right).$$

In other words, $\Delta x_a < \|\Delta x_b\|\infty 1$ and $\Delta x_c \sim \|\Delta x_b\|\infty 1$. For example, a uniform perturbation in echelon 2 produces larger voltage changes in all of echelons 3 and 4 while a uniform perturbation in echelon 3 produces larger voltage changes in echelon 4 but smaller voltage changes in echelon 2. The general conclusion is that a uniform perturbation in an entire echelon is attenuated as it propagates toward the PV buses but is amplified as it moves away from the PV buses. It should be recognized that the result is valid for some ΔQ_b (which depends on the g_{ik} and θ_{ik}) as long as H is a nonsingular M matrix. Note that, although a uniform perturbation in an entire echelon is severe, the results are larger or smaller than the change in voltage in the perturbed echelon.

To clarify the role of the second echelon in disturbance propagation, consider disturbances limited to the second echelon and write

$$\left[\bar{B}_{22} - \bar{C}_{22} - \hat{B}_{23}(\hat{B}_{33} - \hat{C}_{33})^{-1} \hat{B}_{32}\right] \Delta x_2 = -\Delta Q_2 \quad (42)$$

where $\hat{3}$ corresponds to echelons 3 and 4 combined. The row sums of the matrix in (42) are determined by the transmission lines connecting echelon 2 to the PV buses, the shunts in echelon 2 and the shunts reflected from the lower echelons. It is reasonable to assume that the matrix in (42) has positive row sums or is diagonally dominant. Hence, the proof in [6] of the tierwise spreading of disturbances in the active power problem is appropriate to disturbances and iters which are limited to the second echelon. That is, the reactive power problem behaves like the active power problem in disturbance propagation on buses with positive row sums. It is only when negative row sums are encountered that voltages away from the disturbance can change more than those at the disturbance.

It is also possible within restrictions of a nonsingular M matrix to have voltage gain for a single disturbance. To demonstrate this fact, consider a disturbance ΔQ_j at bus j and suppose the Δx_j is larger than Δx_m, $m \neq j$. We will show that the capacitance at some other bus c_{ii} can be increased until $\Delta x_m > \Delta x_j$ without violating the conditions of a nonsingular M matrix. Using (39) we compute the value of $\bar{c}_{ii}$ so that $x_{mj} > x_{jj}$

$$\bar{c}_{ii} > \frac{1}{x_{ii} + x_{ij} \dfrac{x_{mi} - x_{ji}}{x_{jj} - x_{mj}}}. \quad (43)$$

Recall that the condition for H to be a nonsingular M matrix is that $\bar{c}_{ii} < 1/x_{ii}$. We have assumed that $x_{jj} > x_{mj}$ so it is only necessary to select i so that $x_{mi} > x_{ji}$ to make the second term in the denominator positive. If there is no such i, then we are assured that we can reverse the argument and provide voltage gain from bus m to bus j. The value of $\bar{c}_{ii}$ computed from (43) will turn out to be large if both j and m are in echelon 2. However, if j is in echelon 2 and m is in a lower echelon, then the capacitor values are not unreasonable.

VI. Conclusion

In this paper we analyze the justification of the assumption of localized voltage response to changes in reactive power injections. This assumption is often made for designing control schemes and computing algorithms in large scale electric power systems. Here we show that it is not possible to give an exclusive answer to the question of the localized response in the reactive power–voltage problem. The answer is network and operating point dependent. We state (sufficient) mathematical conditions under which a localized response is true. Most of the transmission lines of a power network possess the no-gain property only in a subset of R^2 Although the above results explicitly define regions where a transmission line has a localized response, they cannot be used to demonstrate a voltage gain. Therefore, we develop algebraic type statements to prove that a system may have response which is systemwide.

Next, an "echelon" structure is defined for the linearized equation describing the incremental voltage magnitudes in terms of the incremental reactive injections. By including the capacitive shunts, it is shown that there are a class of "uniform" perturbations (uniform voltage changes in an echelon) that are attenuated in the direction of the PV buses but which are amplified in the direction away from the PV buses. Under modest assumptions concerning the strength of the ties to the PV buses, it is shown that disturbances that are limited to echelon 2 spread in the tier-wise manner previously established for the active power problem.

Acknowledgment

The authors are thankful to the reviewer whose input has contributed to a considerable improvement of the original paper.

References

[1] C. Barbier and J.-P. Barret, "An analysis of phenomena of voltage collapse on a transmission system," *RGE*, vol. 89, no. 10, pp. 673–690, 1980.

[2] J.-P. Barret, P. Bornard, G. Pioger, P. Esclaugon, G. Blanchon, C. Noe and G. Simonnet, "Power system voltage regulation," *RGE*, pp. 37–48, July 1980.

[3] J. Zaborszky, G. Huang, and S. Y. Lin, "Emergency operation of large interconnected power system by decision and control," Dep. Syst., Sci., Math., Washington Univ. St. Louis, MO, SSM Rep. 8304.

[4] J. Zaborszky, G. Huang, and S. Lin, "Control of reactive power and voltage in emergencies," presented at the CIGRE-IFAC Symp. Florence, Italy, 1983.

[5] M. Lanby, T. Mirolinnas, and N. Reppen, "Contingency selection of branch outages causing voltage problems," presented at the 1983 PES Summer Meet., Los Angeles, CA, Pap. 83SM340-347.

[6] N. Narasimhamurthi and M. Ilić-Spong, "A proof of localized response of large interconnected power systems in steady state," in *Proc. All. conf.*, pp. 997–1002, Oct. 1980.

[7] J. Zaborszky, K. W. Whang, and K. V. Prasad, "Fast contingency evaluation using concentric relaxation," *IEEE Trans. Power App. Syst.*, pp. 28–36, Jan./Feb. 1980.

[8] M. Ilić-Spong, "A complex power–complex voltage SE graph for electric power systems studies," in *Proc. 5th Int. Symp. on Network Theory*, Sarajevo, Yugoslavia, Sept. 4–7, 1984, pp. 189–194.

[9] M. Ilić-Spong and J. Zaborszky, "A different approach to load flow," *IEEE Trans. Power App. Syst.*, vol. 101, pp. 168–179, Feb. 1982.

[10] A. Willson, "The no-gain property for networks containing three-terminal elements," *IEEE Trans. Circuits Syst.*, vol. 22, pp. 678–687, 1974.

[11] L. Chua, Y. Lam and K. Stromsmoe, "Qualitative properties of resistive networks containing multi-terminal elements: No-gain properties," *IEEE Trans. Circuits Syst.*, vol. 24, no. 3, 1977.

[12] M. Ilić-Spong, M. Spong, and R. Fischl, "The no-gain theorem and localized response for the decoupled $P-\theta$ network with active power lossess included," *IEEE Trans. Circuits Syst.*, vol. 32, pp. 170–177, Feb. 1985.

[13] H. B. Puttgen and R. L. Sullivan, "A novel comprehensive approach to power system sensitivity analysis," presented at the IEEE Summer Power Meet., Pap. A78-525-8, 1978.

[14] R. Palmer, R. Burchett, H. Happ, and D. Vierath, "Reactive power dispatching for power system voltage security," presented at 1983 IEEE PES Summer Meet., Los Angeles, CA, Pap. 83SM341-345.

[15] H. Happ and K. A. Wirgau, "Static and dynamic VAR compensation in system planning," *IEEE Trans. Power App. Syst.*, vol. PAS-97, Sept./Oct. 1978.

[16] R. J. Kaye and F. Wu, "Analysis of linearized decoupled power flow approximations for steady state security assessment," Univ. California, Berkeley, ERL Memo. UC8/ERL M82/71, 1982.

[17] D. Schulz, "Error in the $Q-V$ decoupled load flow," master degree thesis, Cornell Univ., Ithaca, NY, June 1984.

[18] J. S. Thorp, M. Ilić-Spong and D. Schulz, "Existence and uniqueness of a solution to the reactive power problem for an electric power network," presented at Int. Circuits and Syst. Conf., Montreal, Que., Canada, May 1984.

[19] M. Winokour and B. J. Cory, "Identification of strong and weak areas for emergency state control," presented at IEEE/PES Winter Meet., Dallas, TX, Jan. 29–Feb. 3, 1984, Pap. 84WM 022-0.

[20] A. Berman and R. J. Plemmons, *Nonnegative Matrices in the Mathematical Sciences.* New York: Academic, 1979.

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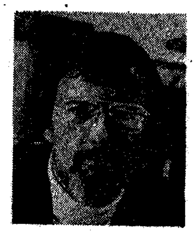

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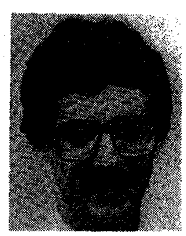

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