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A Structure-based Modeling and Control of Electric Power Systems

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Reactive power/voltage regulation under slow load changes

Key Words—On-line control of electric power systems; structural modeling and hierarchical control of large-scale systems; large-scale dynamic systems; secondary voltage control; tertiary voltage control; slow load variations.

Abstract—This paper addresses the modeling and closed-loop control of large-scale electric power systems operating under normal conditions. A new structure-based modeling approach is introduced—an approach which reveals fundamental properties of power system dynamics, and offers physical insight into the interactions between its constituent subsystems. These properties and insights lead to the formulation of simple, higher level models which represent slow interactions among subsystems driven by slow load variations. The models which evolve from this structure-based approach enable hierarchical control design, without any *a priori* assumptions regarding the strength of the interactions among the subsystems. New control designs are thus proposed at both the subsystem and the interconnected system levels, a key element in their formulation being that both react to variables reflecting interactions between subsystems. Reactive power/voltage (Q/V) control problems are used to illustrate the proposed modeling and control approach. © 1997 Elsevier Science Ltd.

1. INTRODUCTION

In this paper, we propose a structure-based approach to the modeling and control of large-scale electric power systems. Our structure-based approach is illustrated by revisiting the reactive power/voltage control in response to slow deviations in system load.

We suggest two types of models: a decentralized secondary level model for voltage regulation, and an aggregate interconnected system (tertiary) level model for (reactive power) tie-line flow scheduling.

The first model involves regulating voltage according to a chosen performance criterion at each subsystem level, without relying on interactions with the neighboring subsystems. To achieve this, a decentralized secondary-level model is defined. This secondary-level model is expressed in terms of intra-area coupling variables only, while the inter-area variables from the neighboring subsystems are assumed to be measured, known quantities. We show that, by means of such a model, an entirely decentralized control scheme for voltage regulation is possible, without requiring the assumptions typically made in large-scale dynamical systems regarding the strength of interconnections among the subsystems. Secondary-level voltage regulation can be viewed as a stabilization problem, in which the main task is to bring the reactive power tie-line flows as close as possible to their agreed-upon values, while maintaining the voltage within specifications. This model is essential for improved control in the newly evolving utility environment, which often does not allow for the weak interconnection assumptions.

The second model involves the scheduling of reactive power flows between subsystems. The actual scheduling of these tie-line flows is presently done according to decentralized negotiations between the subsystems. The aggregate modeling approach proposed herein is found to be useful in posing tie-line scheduling as a regulation problem at the interconnected system (tertiary) level. The resulting regulation problem is, in turn, well suited for optimizing a performance criterion that captures both technical and economic performance. Because of the relative simplicity of this aggregate model, the optimal scheduling of interconnection variables to meet the system-wide performance criterion can be designed.

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2. REAL POWER/FREQUENCY (P/f) AND REACTIVE POWER/VOLTAGE (Q/V) CONTROL REVISITED

As shown in Fig. 1, an electric power system (of arbitrary topology) can be thought of as a group of locally controlled generators which are interconnected between themselves and to loads through a transmission network.

The local generator controllers are: (i) governors controlling mechanical power output deviations P_{mech} in response to frequency deviations ω around the set values $\omega_G^{ref}[k]$, $k = 0, 1, \ldots$; and (ii) excitation systems controlling field voltage deviations e_{fd} in response to terminal voltage deviations V_G from the set values $V_G^{ref}[k]$, $k = 0, 1, \ldots$. The relevant output variables on the generator side affecting the transmission network and the loads are real power P_G, reactive power Q_G, frequency ω_G, and terminal voltage V_G. P_L, Q_L, ω_L, V_L are corresponding variables at the load side. During the normal operating conditions, governors and excitation systems respond automatically to fast load fluctuations.

Turbine-generator sets, in turn, introduce their own dynamics in producing P_G and Q_G, which, when combined with the governor and excitation systems' dynamics, form what is referred to in this paper as the (local) primary dynamics of the governor–turbine–generator sets. For the frequency ranges associated with normal operating conditions, the network is modeled as an algebraic constraint imposed on the generator and load outputs.

Most uncertainty is seen in load models. These are typically modeled as sinks of constant real power, P_L and reactive power Q_L. Although more realistic models include dependence on their voltage V_L and frequency ω_L, these models are not currently used for the on-line control of an interconnected system. In the approach taken in this paper, loads are modeled as (piecewise) constant power sinks consisting of two components: one component, the statistics on which are known for nominal conditions, is superimposed on another component accounting for load fluctuations of different frequencies. In other words, the postulated load model is

$$P_L = P_L[KT_t] + P_L[kT_s], \quad k, K = 0, 1, \ldots, \quad (1)$$

and

$$Q_L = Q_L[KT_t] + Q_L[kT_s], \quad k, K = 0, 1, \ldots. \quad (2)$$

The $P_L[KT_t]$ and $Q_L[KT_t]$ of (1) and (2) correspond to the real and reactive power load changes over an entire day, for example, and could be updated every T_t interval; $P_L[kT_s]$ and $Q_L[kT_s]$ represent load changes much slower than the largest time constants of primary dynamics on generators. In designing frequency and voltage controls the load fluctuations around the nominal are modeled as unknown disturbances $d[kT_s]$.

It is further assumed in this paper that the real/reactive power decoupling assumption is valid, i.e. that governor actions only affect frequency/real power changes and that excitation systems affect reactive power and voltages only. This assumption is commonly made for studies of load frequency control/automatic generation control (LFC/AGC) under normal operating conditions.

The main purpose of the primary frequency and voltage controllers is to cancel the effects of fast load fluctuations. The system-wide secondary controllers are designed to control frequency and voltage under the slower load changes $P_L[k]$ and $Q_L[k]$. (For notational simplicity in the remainder of this paper KT_t is replaced by K and kT_s by k). Both frequency and voltage controllers are needed to maintain the quality of the frequency and to keep voltages within the desired specifications. The tertiary controller is intended to update settings at secondary controllers in response to slow nominal load changes over the time horizon T_t. In the spirit of the control hierarchy of the proposed approach, the tertiary level sampling time T_t should strongly depend on the rate at which nominal loads are changing. In most of the U.S.A., control centers collect such samples every 15 minutes. The choice of sampling times T_s and T_t should be made according to the load characteristics in the system of interest; thus, they could vary from system to system.

In this paper only reactive power/voltage (Q/V) structure-based modeling and control is described. For the real power/frequency (P/f)

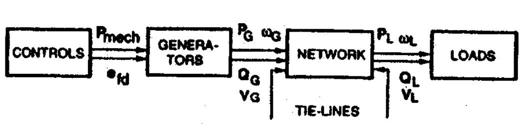


Fig. 1. Basic structure of an electric power system.

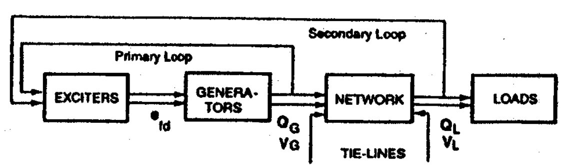


Fig. 2. Typical structure of reactive power/voltage control at a subsystem level.

situation, see Ilić (1994), Eidson and Ilić (1995a, b), and Ilić and Liu (1995).

3. VOLTAGE DYNAMICS AT THE PRIMARY LEVEL

Primary voltage dynamics arise in response to generator excitation system controls and electromagnetic changes at the generator terminals. A natural, structure-based approach to modeling these dynamics is to define a set of (local) state variables which fully specify a single generator's local dynamics, and initially treat the reactive power Q_G demanded by the generator's network connection as an exogenous variable.

The dynamics of all generators are interconnected via a transmission network. The network imposes an algebraic constraint on the total reactive power injection from the generator into the interconnected system, as shown in Fig. 2.

Included in the primary voltage dynamic description is a model of the generator's excitation system. The excitation system controls the field voltage input e_{fd} of a generator in response to the local error signal between the measured generator terminal voltage and the given setting. The goal of primary level excitation control is to maintain the generator terminal voltage at a specified setting. (At present proportional-integral (PI) regulators are commercially available.) As a byproduct of its actions, the excitation system regulates the reactive power Q_G outputted by the generator in response to terminal voltage deviations V_G from a resetable reference value $V_G^{ref}[k]$, $k=0,1,\ldots$. In our primary dynamics model, we treat $V_G^{ref}[k]$ as a (slow) control input.

Various, different models are used to represent primary voltage dynamics. This variability arises from two sources: the choice of excitation system (many exist, each having its own model), and the modeling assumptions made about a generator's electromagnetic dynamics. Despite these differences, the overall structure for a generator's primary dynamics is much the same, regardless of the model. Keeping this in mind, and seeking to streamline our descriptions, we choose to illustrate our structure-based techniques using one of the simpler models of generator voltage dynamics. Standard notation from Sauer and Pai (1993) is used throughout the forthcoming model description.

We commence by assuming $e'_d \approx 0$; in other words, we neglect the effects of damper winding on generator dynamics. The generator voltage dynamics are thus

$$T'_{d0}\dot{e}'_q = -e'_q - (x_d - x'_d)i_d + e_{fd}, \qquad (3)$$

and the excitation system dynamics is

$$T_e\dot{e}_{fd} = -(K_e + S_e)e_{fd} + v_R, \qquad (4)$$

$$T_a\dot{v}_R = K_a r_f - \frac{K_a K_f}{T_f} e_{fd} - v_R - K_a(V_G - V_G^{ref}[k]), \qquad (5)$$

$$T_f\dot{r}_f = -r_f + \frac{K_f}{T_f} e_{fd}. \qquad (6)$$

An important element in these descriptions is i_d, the reactive current component out of a generator. Although not expressed as such, the current component i_d is also a function (through the network interconnection constraints) of the states.

Before proceeding further, let us establish the means of expressing the dynamics of (3)–(6) in a more compact form. At the generator level, introduce the notation (this approximation requires $e'_d \approx 0$)

$$x = [e'_q e_{fd} v_R r_f]^T \approx [V_G e_{fd} v_R r_f]^T. \qquad (7)$$

With this notation, the dynamics in (3)–(6) can be re-expressed in the state-space form:

$$\dot{x}_{LC} = A_{LC}x_{LC} + c_{vec}i_d + b_{vec}V_G^{ref}[k], \quad k=0,1,\ldots, \qquad (8)$$

where the matrix A_{LC} and the vectors b_{vec} and c_{vec} follow directly from (3)–(6). A_{LC} is the system matrix for each generator. Its properties determine the voltage response of each generator to its reactive power demand, represented here by i_d.

The next logical step is to investigate the primary dynamics of a few interconnected generators. However, before we can connect the generators together, network constraints must be introduced.

3.1. *Network constraints*

Network constraints are typically expressed using nodal-type equations which require the complex-valued power into the network $\hat{\underline{S}}^N$ to be equal to the complex valued power $\hat{\underline{S}} = \underline{P} + j\underline{Q}$ injected into each node:

$$\hat{\underline{S}}^N = \mathrm{diag}(\hat{\underline{V}})\hat{Y}^*_{bus}\hat{\underline{V}}^* = \hat{\underline{S}}, \qquad (9)$$

where $\hat{\underline{S}}^N = \underline{P}^N + j\underline{Q}^N$ is the vector of net complex power injections into to all nodes, and $\hat{Y}_{bus}$ is the admittance matrix of the network. $\hat{\underline{V}} = [V_1 e^{j\delta_1}, V_2 e^{j\delta_2}, \ldots]$ is the vector of all nodal voltage phasors, each with magnitude V_i and phase δ_i (collectively, with magnitude $\underline{V}$ and phase $\underline{\delta}$). This equation determines a relation for real power,

$$\underline{P}^{\mathrm{N}} = \underline{P}^{\mathrm{N}}(\underline{\delta}, \underline{V}), \tag{10}$$

and a relation for reactive power

$$\underline{Q}^{\mathrm{N}} = \underline{Q}^{\mathrm{N}}(\underline{\delta}, \underline{V}). \tag{11}$$

Here we discuss only reactive power relations.

Making the definition

$$\underline{i}^{\mathrm{N}} \triangleq \left[\frac{Q_1^{\mathrm{N}}}{V_1} \ \frac{Q_2^{\mathrm{N}}}{V_2} \ \cdots\right]^{\mathrm{T}}, \tag{12}$$

where Q_k^{N} denotes the reactive power flow into network node k, and V_k denotes the terminal voltage at node k.

Let $\underline{i}_{\mathrm{G}}^{\mathrm{N}}$ be the normalized reactive power flows into a subsystem network from generators, and $\underline{F}_{\mathrm{G}}$ the normalized tie-line flows (from other subsystems) into the same network nodes. Also denote by $\underline{i}_{\mathrm{L}}^{\mathrm{N}}$ the reactive power flow out of the network into loads, and by $\underline{F}_{\mathrm{L}}$ the tie-line flows (from other subsystems) into these load nodes. Using these assignments, (12) can be partitioned further into normalized reactive power network constraints at the generator terminals

$$\underline{F}_{\mathrm{G}} + \underline{i}_{\mathrm{d}} = \underline{i}_{\mathrm{G}}^{\mathrm{N}}(\underline{\delta}, \underline{V}), \tag{13}$$

and similar constraints at the load terminals

$$\underline{F}_{\mathrm{L}} - \underline{i}_{\mathrm{L}} = \underline{i}_{\mathrm{L}}^{\mathrm{N}}(\underline{\delta}, \underline{V}). \tag{14}$$

Under the real-reactive power decoupling assumption $\partial \underline{Q}^{\mathrm{N}}/\partial\underline{\delta} = 0$. Since the terminal voltages serve only as normalizing factors, this decoupling assumption is equivalent to $\partial \underline{i}^{\mathrm{N}}/\partial\underline{\delta} = 0$. Linearizing (13) and (14) under this decoupling assumption and then rearranging yields

$$\begin{bmatrix} \underline{i}_{\mathrm{d}} \\ -\underline{i}_{\mathrm{L}} \end{bmatrix} = \begin{bmatrix} J_{\mathrm{GG}} & J_{\mathrm{GL}} \\ J_{\mathrm{LG}} & J_{\mathrm{LL}} \end{bmatrix} \begin{bmatrix} \underline{V}_{\mathrm{G}} \\ \underline{V}_{\mathrm{L}} \end{bmatrix} - \begin{bmatrix} \underline{F}_{\mathrm{G}} \\ \underline{F}_{\mathrm{L}} \end{bmatrix}, \tag{15}$$

where $\underline{V}_{\mathrm{L}}$ indicates the terminal voltages at load buses, and $J_{jk} = \partial \underline{i}_j^{\mathrm{N}}/\partial \underline{V}_k$, j, $k = \mathrm{G}, \mathrm{L}$. Observe that, upon linearization, the variables appearing in (15) are all incremental quantities: their values indicate offsets from the nominal value at which the linearization occurred.

Assume J_{LL} to be invertible under normal operating conditions. Define

$$C_{\mathrm{V}} = -J_{\mathrm{LL}}^{-1} J_{\mathrm{LG}}. \tag{16}$$

Solving (15) for $\underline{V}_{\mathrm{L}}$ yields

$$\underline{V}_{\mathrm{L}} = C_{\mathrm{V}} \underline{V}_{\mathrm{G}} + J_{\mathrm{LL}}^{-1}(\underline{F}_{\mathrm{L}} - \underline{i}_{\mathrm{L}}), \tag{17}$$

which is an important relationship, as it specifies the dependence of load voltages on generator voltages and load reactive power disturbances.

Next, solve (15) for $\underline{i}_{\mathrm{d}}$, which provides another important result:

$$\underline{i}_{\mathrm{d}} = K_{\mathrm{Q}} \underline{V}_{\mathrm{G}} + \underline{F} - D_{\mathrm{Q}} \underline{i}_{\mathrm{L}}, \tag{18}$$

where

$$K_{\mathrm{Q}} = J_{\mathrm{GG}} + J_{\mathrm{GL}} C_{\mathrm{V}}, \tag{19}$$

$$D_{\mathrm{Q}} = -J_{\mathrm{GG}} J_{\mathrm{LL}}^{-1}, \tag{20}$$

$$\underline{F} = D_{\mathrm{Q}} \underline{F}_{\mathrm{L}} + \underline{F}_{\mathrm{G}}. \tag{21}$$

3.2. *Generator interconnection to form a subsystem*

With (18), the interconnection variable i_{d} need no longer be treated as an exogenous variable: generators can now be attached to the network, and dynamics arising from their intrasubsystem interconnection can be studied.

Consider, within an interconnected system, a single area (subsystem) with m generators, each of them having the local primary dynamics introduced in (18). The state-space formulations for the linearized voltage dynamics (8) of each and every generator in the subsystem, along with the interconnection constraint (18), form a closed-loop model for the primary dynamics of this subsystem.

To arrive at this model, a few definitions are required. First, the definitions of states and controls:

$$\underline{x} = \begin{bmatrix} x_{\mathrm{LC}}^1 \\ \vdots \\ x_{\mathrm{LC}}^m \end{bmatrix}, \quad \underline{V}_{\mathrm{G}}^{\mathrm{ref}}[k] = \begin{bmatrix} V_{\mathrm{G}_1}^{\mathrm{ref}}[k] \\ \vdots \\ V_{\mathrm{G}_m}^{\mathrm{ref}}[k] \end{bmatrix}, \tag{22}$$

where each vector of the generator states, x_{LC}^i, is as defined as in (7), and each scalar voltage reference, $V_{\mathrm{G}_i}^{\mathrm{ref}}$ is as described in (6).

The mechanisms by which $\underline{V}_{\mathrm{G}}$ (which appears in (18)) is extracted from the set of all states $\underline{x}$ involves using $V_{\mathrm{G}}^i = E\underline{x}$, where $E = \text{block diag}\,(e, e \ldots, e)$, and $e = [1 \ \ 0 \ \ 0 \ \ 0]$.

Lastly, the matrices used in the final state-space description are defined as:

$$A = \text{block diag}\, A_{\mathrm{LC}}^i \quad i = 1, \ldots, m, \tag{23}$$

$$\bar{B} = \text{block diag}\, b_{\mathrm{vec}}^i \quad i = 1, \ldots, m, \tag{24}$$

$$\bar{C} = \text{block diag}\, c_{\mathrm{vec}}^i \quad i = 1, \ldots, m. \tag{25}$$

One applies these definitions to build a composite state space from the stand-alone (local) generator state-space descriptions (7), and then uses the interconnection constraint (18) to couple the generators. The resulting subsystem model for primary dynamics is found to be

$$\dot{\underline{x}} = (A + \bar{C} K_{\mathrm{Q}} E)\underline{x} + \bar{B} \underline{V}_{\mathrm{G}}^{\mathrm{ref}}[k] - \bar{C}\underline{F} + \bar{C}\underline{i}_{\mathrm{L}}. \tag{26}$$

Note that the interconnection constraints do not add to the total number of states required by this description. This is the first major difference between the voltage and frequency dynamical models: the analogous expression for primary-level real power dynamics requires a $\underline{P}_G$-extended state-space description (Liu, 1993; Ilić, 1994; Eidson and Ilić, 1995; Ilić and Liu, 1995).

4. VOLTAGE DYNAMICS AT THE SECONDARY LEVEL

Typical excitation system designs yield stable closed-loop primary level voltage dynamics which are fast relative to the update time T_s of the secondary-level control $V_G^{ref}[k]$. This separation in time scales between the primary- and secondary-level dynamics is called *vertical separation*; an implied, necessary condition for such vertical separation is that the primary dynamics are stable (so they settle before secondary control is invoked).

The function of secondary-level control—for the reactive power/voltage application—is to regulate load voltages at pilot points. Provided vertical separation holds, secondary control achieves this function by changing the reference settings for generator terminal voltages.

4.1. *A secondary level model for load voltage regulation*

Assume that vertical separation holds. Under this assumption, subsystem primary dynamics settle to a steady-state value before any new secondary control action is invoked. The role of secondary voltage control is to manipulate voltage reference settings so that, in the face of load disturbances, certain subsystem states eventually settle at desirable operating points. Each subsequent change in secondary controls—a change which occurs every T_s seconds—results in a different steady-state operating point; thus, the secondary controller establishes the sequence of moving equilibria to which the primary dynamics successively settle.

We use the principle of vertical separation to develop the secondary-level system model. This model construction involves first finding the steady-state solution for a certain state space of the primary dynamics, in this case $\underline{V}_G$, and relating it to the controls, $\underline{V}_G^{ref}[k]$. This is effected by setting the derivatives in the primary-level state-space description to zero, and solving for the steady-state $\underline{V}_G$ in terms of $\underline{V}_G^{ref}[k]$.

Typically, the steady-state solution is more easily found by starting at the (local) single-generator level and building. The exact procedure to find V_G is as follows:

(i) Utilize (3)–(6) to solve for the relationship between $V_G[k]$, $V_G^{ref}[k]$, and the interconnection constraint i_d (for a single generator):

$$(x_d - x_d')i_d[k] = \frac{K_a}{K_e + S_e} V_G^{ref}[k] - \left(1 + \frac{K_a}{K_e + S_e}\right)V_G[k]. \tag{27}$$

Loosely speaking, (27) is an expression analogous to the generator droop characteristic found in frequency regulation (Zaborszky, and Rittenhouse, 1969).

(ii) Aggregate the parallel results from other generators to form vector relations:

$$X_{dd'}\underline{i}_d[k] = K_{mat}\underline{V}_G^{ref}[k] - (I - K_{mat})\underline{V}_G[k], \tag{28}$$

where $X_{dd'}$ and K_{mat} are diagonal matrices. (Call $\underline{V}_G^{ref}[k] = (I + K_{mat}^{-1})\underline{V}_G[k]K^{-1}X_{dd'}\underline{i}_d$ (which is obtained by algebraically manipulating (28)), the (multi-machine) *voltage droop characteristic.* Note that if $K^{-1} \equiv 0$, i.e. the parameter ratio $(K_e + S_e)/K_a \equiv 0$ for every generator, then $\underline{V}_G[k] = \underline{V}_G^{ref}[k]$ for all k. We call this characteristic a *uniform droop characteristic.*)

(iii) Eliminate $\underline{i}_d[k]$ in (28) by substituting the linearized network constraint (18) into (28), and then solve for $\underline{V}_G[k]$:

$$\underline{V}_G[k] = (I + K_{mat} + X_{dd'})^{-1}\{K_{mat}\underline{V}_G^{ref}[k] - X_{dd'}\underline{F}[k] + X_{dd'}D_Q\underline{i}_L[k]\}. \tag{29}$$

The goal was to construct a model relevant for load (not generator) voltage control. The foundation for this load voltage model is found by inserting (29) into (17), yielding

$$\underline{V}_L[k] = HK_{mat}\underline{V}_G^{ref}[k] + (HX_{dd'}D_Q - J_{LL}^{-1})\underline{i}_L[k] - (HX_{dd'}D_Q - J_{LL}^{-1})\underline{F}_L[k] - HX_{dd'}\underline{F}_G[k], \tag{30}$$

where $H = C_V(I + K_{mat} + X_{dd'}K_Q)^{-1}$.

Evaluating (30) at k and $k+1$ and then taking the difference produces the secondary-level voltage model:

$$\begin{aligned}\underline{V}_L[k+1] - \underline{V}_L[k] = {} & U(\underline{V}_G^{ref}[k+1] - \underline{V}_G^{ref}[k]) \\ & + M_L(\underline{F}_L[k+1] - \underline{F}_L[k]) \\ & - M_G(\underline{F}_G[k+1] - \underline{F}_G[k]) \\ & - M_L(\underline{i}_L[k+1] - \underline{i}_L[k]),\end{aligned} \tag{31}$$

where $U = HK_{mat}$, $M_L = HX_{dd'}D_Q - J_{LL}^{-1}$, and

$M_G = HX_{dd'}$. The fairly simple structure of model (31) is fundamental to the development of decentralized secondary-level voltage controllers, and also to the tertiary-level controllers which coordinate them.

Secondary-level control is called automatic voltage control (AVC) in France and Italy (Carpasso *et al.*, 1980; Paul and Léost, 1986; Paul *et al.*, 1987). The main function of AVC is to respond to reactive load disturbances $\underline{d}[k] = \underline{Q}_L[k+1] - \underline{Q}_L[k]$ (which, given our formulation, corresponds to reacting to the disturbances $\underline{i}_L[k+1] - \underline{i}_L[k]$). AVC is implemented on generator units the voltage set points $\underline{V}_G^{ref}[k]$ of which are automatically changed to respond to deviations in load voltages $\underline{V}_L[k]$ at a chosen subset of loads, called *pilot point loads*. The associated pilot load voltages are selected from the full set of load voltages by the relation

$$\underline{V}_p[k] = C\underline{V}_L[k], \qquad (32)$$

where C is a matrix of ones and zeros.

4.2. *Secondary level voltage control*

AVC should be designed to keep operation of subsystems as autonomous as possible. The prime objectives of secondary voltage control are to:

(i) reschedule the controls $\underline{V}_G^{ref}[k]$ of each subsystem so that subsystem reactive load deviations $\underline{Q}_L[k]$, $k = 0, 1, \ldots,$ are met;

(ii) regulate pilot voltages $\underline{V}_p[k]$ to a (tertiary-level control-determined) set point of $\underline{V}_p^{ref}[K]$ (where $KT_t \gg kT_s$);

(iii) maintain $\underline{F}_L[k] \approx 0$ and $\underline{F}_G[k] \approx 0$ as long as (reactive) reserves within each area are available (area control principle); and

(iv) optimize subsystem performance (in terms of total reactive reserve or total transmission losses).

4.2.1. *Controllability at the secondary level.* As we shall demonstrate shortly, the use of pilot load voltages for secondary control is a practical necessity.

Claim 4.1. The dynamic system given in (31) is not, in general, fully controllable.

Proof. The controllability matrix for (31) is simply U, which, in terms of its constituent elements, can be written as

$$U = C_V(I + K_{mat} + X_{dd'}K_Q)^{-1}K_{mat}. \qquad (33)$$

K_{mat} is diagonal; however, the sensitivity matrix C_V has dimension $n_l \times n_g$, where n_l (here) designates the number of load buses and n_g designates the number of generators in a subsystem. The rank of U is at, best, $\min(n_l, n_g)$. Therefore, if $n_l > n_g$, then $\text{rank}\{U\} \le n_g < n_l$, which implies that not all n_l of the loads can be controlled. In other words, the subsystem is not, in general, fully controllable.

This tells us that, given the usual case, where the number of loads in a subsystem greatly outnumbers the number of generators, not all load voltages can be independently regulated. At best, the same number of loads as generators actively participating in secondary voltage regulation can be fully controlled. This motivates the AVC's use of pilot load voltages V_p, the number of which should not exceed the number of generators in a subsystem. The control-driven model for these voltages can be obtained by combining (31) and (32), resulting in

$$\underline{V}_p[k+1] - \underline{V}_p[k] = CU\underline{u}_s[k] + -CM_G\underline{f}_G[k] + CM_L(\underline{d}_s[k] - \underline{f}_L[K]), \qquad (34)$$

where

$$\underline{u}_s[k] \triangleq \underline{V}_G^{ref}[k+1] - \underline{V}_G^{ref}[k], \qquad (35)$$

$$\underline{f}_L[k] \triangleq \underline{F}_L[k+1] - \underline{F}_L[k], \qquad (36)$$

$$\underline{f}_G[k] \triangleq \underline{F}_G[k+1] - \underline{F}_G[k], \qquad (37)$$

$$\underline{d}_s[k] \triangleq \underline{i}_L[k+1] - \underline{i}_L[k]. \qquad (38)$$

The dynamics in model (34) are the consequence of defining $u_s[k]$ as an *increment* of control settings, rather than as actual settings. The secondary-level control $u_s[k]$ is, therefore, truly an integral controller.

4.2.2. *Secondary level control design.* Currently, secondary voltage regulators are based on a model in which the flow changes $\underline{f}_G[k]$ and $\underline{f}_L[k]$ are neglected. The simple proportional feedback law which conventional voltage regulators use is

$$\underline{u}[k+1] = K_s(\underline{V}_p[k] - \underline{V}_p^{set}[K]), \qquad (39)$$

where $\underline{V}_p^{set}[K]$ is a set-point value for the pilot voltages assigned either externally or by a tertiary-level controller, and $K \equiv KT_t$ indicates the very slow $(T_t \gg T_s)$ update rate for tertiary-level controls.

Under this control law, the closed-loop dynamics at the secondary level are described by

$$\underline{V}_p[k+1] - \underline{V}_p[k] = CUK_s(\underline{V}_p[k] - \underline{V}_p^{set}[K]) + CM_L(\underline{d}_s[k] - \underline{f}_L[k]) - CM_G\underline{f}_G[k]. \qquad (40)$$

As mentioned previously, in order to fully control the pilot load voltages, the pilot points must be selected so that rank$\{C\} \geq m$. The secondary control design problem is then one of choosing an appropriate control feedback gain K_s (Paul *et al.*, 1987); given our formulation, the voltage regulation of pilot point loads can be combined with an optimization with respect to another chosen performance criterion. (K_s is typically designed so that a pilot point voltage settles in 3 minutes (Paul *et al.*, 1987).)

Our intention here, however, is to take one step beyond the design of K_s. We now propose an improved secondary voltage controller which *does* take into account the effect of the flow changes, yet still remains decentralized. To compensate fully for the effect of interconnections, consider a new control feedback law of the form

$$\underline{u}_{s2}[k] = K_s(\underline{V}_p[k] - \underline{V}_p^{set}[K]) + G_L\underline{f}_L[k] + G_G\underline{f}_G[k], \quad (41)$$

where K_s is the same as in the conventional secondary controller (39). Since $u_s[k]$ is already an integral controller (defined in terms of increments of control settings), there is no obvious gain from making $u_s[k]$ a PI controller, rather than just proportional. This controller serves the main purpose of changing set points for conventional exciters.

Appropriate choices for the matrices G_L and G_G enable a complete cancellation of the effects of external ('tie-line') flows on pilot voltages. To see how this is possible, first observe that, with the newly proposed secondary control (41), the closed-loop dynamics become

$$\begin{aligned}\underline{V}_p[k+1] - \underline{V}_p[k] = {} & CUK_s(\underline{V}_p[k] - \underline{V}_p^{set}[K]) \\ & + C(UG_L - M_L)\underline{f}_L[k] \\ & + C(UG_G - M_G)\underline{f}_G[k] \\ & + CM_L\underline{d}_s[k]. \end{aligned} \quad (42)$$

From this we see that, provided CU is invertible, the external flows can be eliminated completely by making the following choices for G_L and G_G:

$$G_L = (CU)^{-1}CM_L, \quad (43)$$

$$G_G = (CU)^{-1}CM_G. \quad (44)$$

The benefits of the improved secondary controller are expected to be particularly noticeable for tightly coupled subsystems. Note that, due to controllability issues, only n_g external flows can be fully controlled, where n_g is the number of active controls (generators participating in voltage control) in the given subsystem.

4.3. *Simulation of secondary-level control performance*

In this section, we provide a numerical example comparing the performance of two secondary-level voltage controllers:

(i) a conventional, proportional-feedback controller using the control law given in (39); and

(ii) the newly proposed controller with proportional feedback and tie-line flow compensation (see (41)–(44)).

Simulations were done assuming a small, 9-bus electric power system, like the one depicted in Fig. 3.

Region I consists of buses 1 and 7; the rest is region II. the pilot points are buses 1, 2, and 3. The line parameter data for the 9-bus example is specified in Table 1. The nominal operating point is characterized by the unity voltage magnitude and zero voltage phase angles across the system (so-called flat start operating point). (*Note*: Any other nominal operating point could be used.)

A uniform voltage droop characteristic is assumed, i.e. $\underline{V}_G[k] \equiv \underline{V}_G^{ref}[k]$; this actually occurs when a system has parameters such that $K_{mat}^{-1} \equiv 0$. This simplifying assumption produces system matrices $U = C_V$, $M_L = J_{LL}^{-1}$, and $M_G = 0$.

The load is assumed to have a step increase at bus 5 at $t = 0$; the effect of disturbance is seen in changes of the initial conditions and then it vanishes. The feedback gain K_s, used for both the conventional and tie-line compensating controller, was designed such that the pilot voltages settle exponentially in 3 minutes; more precisely, CUK_s was set equal to λI, where λ is a scalar, and λ selected such that $I + CUK_s$ (which specifies the closed-loop pilot voltage dynamics) is a diagonal matrix with a time constant of 3 minutes.

Figure 4(a) shows the pilot voltage responses using the conventional feedback law. Due to

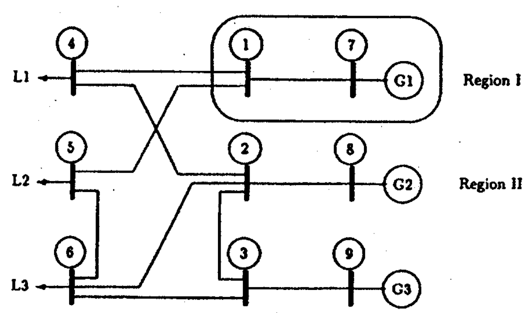


Fig. 3. A 9-bus electric power system.

Table 1. Line parameter and operating point data for the 9-bus example

Line	1–4	1.5	1.7	2.3	2.4	2.6	2.8	3.6	3.9	5.6
b	5	7	7.69	15.67	21.55	7.19	8.33	12.49	10	9.8
a	0	0.15	0	0	1.02	0.34	0	0.31	0	0

tie-line interactions, overshoot occurs and settling time is longer than design specifications.

Figure 4(b) shows the pilot voltage responses with the tie-line compensating control. It is apparent that the addition of $G_G f_G[k]$ to the proportional feedback law improves the system's regulating response by both eliminating overshoot and ensuring prompt settling. This improvement is expected to be significant when tie-lines are strong and meshed.

Figure 5 compares the voltage regulation performance of the two controllers at nonpilot points. Again, the tie-line compensating controller demonstrates an appreciable improvement in performance.

4.4. *Interaction variables and secondary voltage dynamics*

Having now studied and simulated approaches which enable autonomous load voltage control at the subsystem level, we next prepare the framework for another hierarchy of control, i.e. tertiary-level control. The responsibility of tertiary-level control is to coordinate the subsystem controllers, a task which it accomplishes via manipulating the set points $V_{p_i}^{set}[K]$ for every subsystem or region $i = \mathrm{I}, \mathrm{II}, \ldots, R$, in the entire interconnected system. Since the focus of tertiary control is on subsystem coordination, one would expect the key elements in its make-up to be intersubsystem interactions. This is indeed the case; in our structure-based rubric, the quantities that represent such subsystem–subsystem interactions are called *interaction variables* (Liu, 1993).

Let us begin the study of interaction variables by recapitulating (18), which states $\underline{i}_d = K_Q \underline{V}_G + \underline{F} - D_Q \underline{i}_L$. Since $i_d \approx Q_G/V_G$ is intimately related to a generator's reactive power output, scrutiny of (18) tells us that the matrix K_Q reflects the effect of voltage changes on reactive power outputs from generators. This matrix is, in general, structurally nonsingular; only under unusual operating conditions could K_Q become numerically singular.

To introduce the interaction variables relevant

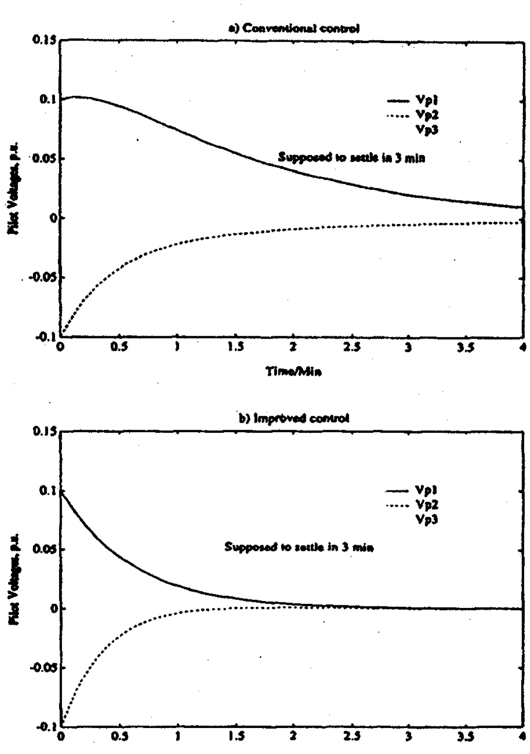


Fig. 4. Pilot voltages: (a) conventional; (b) improved.

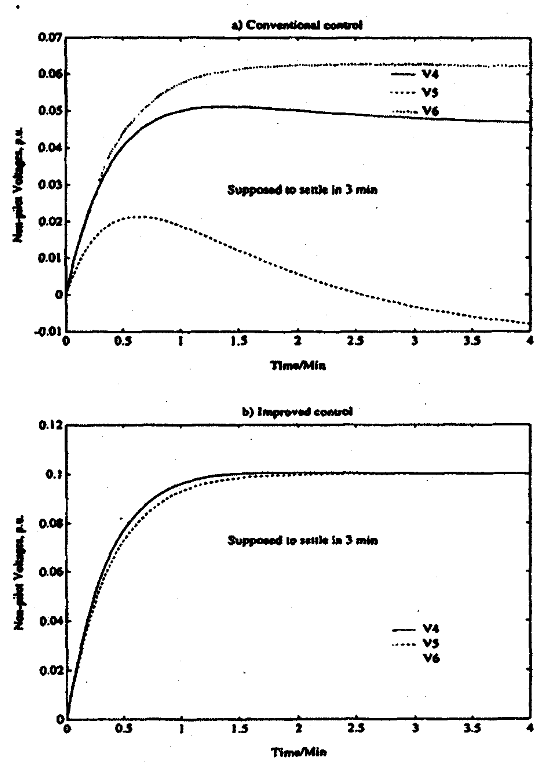


Fig. 5. Nonpilot voltages: (a) conventional; (b) improved.

for controlling reactive power/voltage dynamics, recall that until the number of loads n_l in a subsystem was reduced to (at least) the number of controls n_g, the secondary-level model describing the dynamics of $\underline{V}_L[k]$ (31) was not fully controllable. Another way of interpreting this is that the unreduced model is structurally singular, since the matrix C_V has a maximum rank of n_g. This structural singularity is unique to the voltage because it is caused entirely by a deficient degree of controllability at the subsystem level. (The structural singularity in the frequency/real power problem arises from the fact that all generator frequency states must be referred to a fixed reference, this being one of the generator frequencies in the system. Thus, when all of the frequency states in a subsystem are aggregated, a rank-1 deficiency arises (Liu, 1993; Ilić, 1994).

We now introduce a definition of an interaction variable.

Definition. An interaction variable $\underline{y}$ of any subsystem under study is a variable which satisfies

$$\underline{y}[k+1] = \underline{y}[k] \quad \forall k \tag{45}$$

for any control actions $\underline{u}_s[k]$ when all interconnections of this subsystem with neighboring subsystems are removed and the subsystem is free of disturbances.

Assume a linear transformation T relating $\underline{y}[k]$ to $\underline{V}_L[k]$:

$$\underline{y}[k] = T\underline{V}_L[k]. \tag{46}$$

To determine a value of T which transforms $\underline{V}_L[k]$ into an interaction variable, substitute (46) into (31) and solve for a T which zeros out of the coefficient matrix for the control term $(\underline{V}_G^{ref}[k+1] - \underline{V}_G^{ref}[k])$. One finds from this that a candidate T must satisfy the condition

$$TC_V(I + K_{mat} + X_{dd'}K_Q)^{-1}K_{mat} = 0, \tag{47}$$

which reduces to

$$TC_V = 0, \tag{48}$$

since the quantity $(I + K_{mat} + X_{dd'}K_Q)^{-1}K_{mat}$ is, in general, nonsingular.

Remark 4.1. Note that any T which satisfies (48) would be a matrix of dimension $(n_l - n_r) \times n_l$, where $n_r = \text{rank}\{C_V\}$. The rows of T lie in the left nullspace of C_V.

Remark 4.2. A curious result is that any T which satisfies (48) not only nulls out the control-induced term—it also nullifies the tie-line flows into a subsystem's generators ($\underline{F}_G[k]$). T does not, however, completely cancel the terms associated with reactive load disturbances ($\underline{i}_L[k]$), or tie-line flows into loads ($\underline{F}_L[k]$).

Using the above definition, one can show that the dynamics for voltage interaction variables are expressed by

$$\underline{y}[k+1] = \underline{y}[k] + TJ_{LL}^{-1}(\underline{f}_L[k] - \underline{d}_s[k]). \tag{49}$$

5. COORDINATED VOLTAGE CONTROL OF A MULTIREGIONAL ELECTRIC POWER SYSTEM

The conceptual development and simulation demonstration of the preceding sections indicate that it is possible to introduce decentralized secondary-level voltage regulation at a regional level. Moreover, the enhancement made to secondary-level control guarantees adherence to a regional performance standard, independent of voltage changes in neighboring regions. Such independent, autonomous regional operation is possible as long as:

(i) the matrix C_V, which is operation point specific (see the definition of C_V in (16)), has a rank equal to or exceeding the number of pilot points,

(ii) no constraints on voltage controls are reached; and

(iii) no tripping of secondary units takes place.

Control design becomes qualitatively more complex when system-wide reactive reserves must be rescheduled. Typically done to maintain the reactive power/voltage of a region within constraints, this rescheduling involves changing reactive power tie-line flows between the regions. In essence, changing the tie-line flows allows the system to send aid to regions which, due to constraints, are reactive power deficient.

Two crucial questions could be asked in this context: Is reliance on reactive power/voltage support from electrically distant areas realistic, and, if so, when is such support needed? In many systems, economic scheduling of real power tie-line flows is routinely done so that the system-wide cost of the real power supplied is minimized. Not obvious, however, is a similar concept which would be meaningful and feasible for voltage scheduling, particularly since reactive power has a fundamental property of not 'traveling' far Thorp *et al.* (1986a). Only when voltage constraints begin to be approached would the effect of neighboring regions become significant. Otherwise, the only issue is time constants and the quality of regional response at which the interconnected system would settle.

In principle, one could implement a coordination scheme which adheres to a system-wide performance criterion in either an entirely decentralized setting (at a regional level), or a centralized (interconnected system) level. Decentralized controllers, however, require region-separable performance criteria, an often restrictive requirement (Ilić *et al.*, 1995). To the second approach, centralized coordination, we assign the name tertiary-level control. A new framework for system-wide voltage coordination, its description, and derivation are discussed in this section.

5.1. *Centralized, tertiary level controls*

With the increased trend for large energy transfers over long distances, the problem of maintaining voltages within acceptable operating specifications has emerged in operating and planning power systems throughout the world. The system-wide voltage coordination problem is viewed by many as a specialized OPF problem. There are some drawbacks to the OPF technique. First of all, it does not offer much engineering insight for interpreting numerical solutions. As a consequence, when there are convergence problems, OPF cannot explain if the cause is the lack of solutions or the particular numerical method involved. Also, OPF does not offer much opportunity for handling the specific problems of individual regions. Another drawback is that a large number of data for almost all variables of the entire system are needed in order to carry out the optimization. Therefore, state estimations are necessary for those unmeasured states. Errors in the state estimations can cause problems for the algorithm.

The main purpose of tertiary-level voltage control is to update set values for reactive power tie-line flows $F[K]$, $K=0, 1, \ldots,$ on the tertiary-level time-scale in order to optimize system-wide performance for the anticipated base load $Q_L[K]$, $K=0,1,\ldots$. This could be done on an hourly basis, if not more often, in accordance with the statistical information on base load. The actual setting of tie-line flows is achieved by changing the settings of secondary voltage controllers $V_G^{set}[K]$. Because this is done so infrequently, it could involve recomputing the basic matrices around a new operating point for the anticipated load over the time horizon T_t.

The idea behind the coordination of regional controllers is to establish feasible schemes for maintaining voltages throughout the interconnected system within the prespecified limits, subject to available reactive power resources. Although in this paper only reactive power reserves of generators are of direct interest, the coordination is directly applicable to all other sources of reactive energy that have primary controls responding to local voltages, such as static Var compensators and under-load tap changing transformers.

The determination of the optimal set values for the pilot load voltages is formulated as an optimization problem. However, the notion of *optimal* voltage profiles remains an open research question even for the simplest possible network with one generator supplying power to a single load (Ilić *et al.*, 1994). This is because the optimal operation of the network has not yet been rigorously defined. The main performance candidates are concerned with:

(i) system reactive reserves,

(ii) transmission losses,

(iii) voltage proximity to the prespecified limits, and

(iv) flow scheduling.

We recognize that some performance criteria may be more relevant for normal operating conditions and others for emergency conditions. Therefore, coordination strategies may depend on the system operating mode. In this sense, there will be a certain degree of adaptation to the operating mode. Conventional thinking is that, under normal operation, one wishes to minimize transmission loss, assuming that the system is well within the reactive reserve and voltage limits.

5.2. *Performance criteria*

Assuming secondary-level control is carried out properly, the objective of the tertiary-level control is to determine the values for the pilot load voltages, or, equivalently, the set values for the generator voltages, over the tertiary-level time scale T_t, so that the global system as a whole operates optimally according to a certain performance criterion. In this section, we discuss some general aspects of the performance criteria to be used for the optimization process.

Since the system is composed of three major components (the generators, the transmission network, and the loads) the overall performance criterion can be written as

$$J = J_{gen} + J_{net} + J_{load}, \tag{50}$$

where J_{gen}, J_{net}, and J_{load} are the performance criteria corresponding to each of the three major components of the power system. Specifically, using the above performance criterion, we can achieve the following:

(i) *Generation alignments* to nearly equalize the ratios of actual generation to the maximum capacity of all or part of the generators.

(ii) *Flow scheduling* to schedule tie-line flows among the interconnected regions so that the global system operates in a coordinated fashion.

(iii) *Security enhancement* to ensure that the generators stay within their limits as much as possible.

(iv) *Loss minimization* to reduce the losses on the transmission network.

For the generators, we need to deal with both the reactive power generation and the terminal voltages. Therefore, the choice for the generator performance criterion can be further decomposed as

$$J_{gen} = J_Q + J_V, \quad (51)$$

with J_Q pertaining to reactive power generation and J_V to the voltage limit problem. The term J_Q is introduced to ensure that the reactive power generation remains within physically permissible limits. One simple quadratic form, for example, is

$$J_Q = (Q_G[K] - Q_G^{opt})^T W_Q (Q_G[K] - Q_G^{opt}), \quad (52)$$

where Q_G^{opt} is the desired nominal point inside the limit band of the reactive power generation, and the weighting matrix $W_Q = W_Q^T \geq 0$. The term J_V is primarily to ensure that the generator terminal voltages stay within the allowable bounds. The simple quadratic form for J_V is expressed as

$$J_V = (V_G[K] - V_G^{opt})^T W_u (V_G[K] - V_G^{opt}), \quad (53)$$

where V_G^{opt} is the desired nominal point inside the limit band of the generator terminal voltages, and the weighting matrix $W_u = W_u^T \geq 0$. This kind of performance criterion tends to keep the generator generation outputs and terminal voltages close to their desired nominal values, if heavy weights are given to these terms. The justification for this type of performance criterion is that it can eliminate the situation of some generators hitting their physical operating limits under heavy-load conditions.

Similarly, for the transmission network, we can decompose the performance criterion into a term involving the total losses on the transmission network and a term involving rescheduling the tie-line flows. For the loads, the primary concern is also for the critical pilot node load voltages to stay within acceptable bounds. An expression similar to (53) can be written for the load voltages:

$$J_{load} = (V_p[K] - V_p^{opt})^T W_v (V_p[K] - V_p^{opt}), \quad (54)$$

where V_p^{opt} is the desired point inside the limit band of the load voltages, and the weighting matrix $W_v = W_v^T > 0$. This performance criterion tends to keep the load voltages close to the desired point V_p^{opt}.

In the process of solving the optimal control problem given in (50), constraints among the tie-line flows, unit generation outputs, losses, and the set values for the pilot load voltages on the tertiary-level time-scale are needed. These constraints for the time K involve the values of these quantities at the previous time $(K-1)$. As a consequence, the optimal solution for the time K involves quantities at the previous time $(K-1)$. Therefore, the pilot load voltage settings as a result of the optimal control problem form another discrete-time sequence on the very slow time-scale T_t. The basic requirement for the optimization process is that it must guarantee the stability of this discrete event process.

It is emphasized that this process of tertiary-level control involves only information about the generators participating in secondary-level control and about the pilot point loads, plus the tie-line flows if they are to be rescheduled. The amount of data and computation involved are much less than that needed for full-scale optimal power flow calculation, where information about all loads is necessary.

5.2.1. *Derivation.* Assume the interconnected system consists of R subsystems. Enumerate these systems I, II, ..., R. To enable us to treat the interconnected system as we did the subsystems at the secondary level, assemble a vector of all pilot voltages, from every subsystem, within the interconnected system:

$$\underline{V}_L = \begin{bmatrix} \underline{V}_L^I \\ \underline{V}_L^{II} \\ \vdots \\ \underline{V}_L^R \end{bmatrix}, \quad (55)$$

where $\underline{V}_L^I$ corresponds to the vector of load voltages in Area I.

Now treat this set of states as corresponding to a very large subsystem with no external flows. Apply (34), which specifies subsystem level secondary dynamics, to the new state space (55), and set the external flows $f[K] = 0$. This yields the dynamic description

$$V_L[K+1] - V_L[K] = C_V K_{inv} k_{mat} u[K] + (C_V K_{inv} X_{dd'} D_Q - J_{LL}^{-1}) d[K], \quad (56)$$

where $K_{inv} = (I + K_{mat} + X_{dd'} K_Q)^{-1}$.

In this description,

(i) $u[K]$ and $d[K]$ are vectors containing, respectively, the controls and disturbances corresponding to every generator in the entire interconnected system;

(ii) k_{mat}, K_{inv}, and $X_{dd'}$ are diagonal matrices assembled from their constituent subsystem level components, K^i_{mat}, K^i_{inv}, and $X^i_{dd'}$, $i = \mathrm{I}, \mathrm{II}, \ldots, R$;

(iii) C_V, D_Q, and J_{LL} are interconnected system equivalents of the subsystem level C^i_V, D^i_Q, and J^i_{LL} (but due to the inclusion of interconnections, these matrices are not block diagonal);

(iv) the time indices K correspond to tertiary level sampling instants KT_t, $K = 0, 1, 2, \ldots$.

In general, not all generators are involved in voltage control. Designate the pared-down vector of controls corresponding to participating generators u_p. To remove the generators which do not participate, introduce a matrix C_p composed of ones and zeros (C_p has, at most, one nonzero entry per row), which selects from u only the controls corresponding to u_p:

$$u_p[K] = C_p u[K]. \quad (57)$$

Due to the unique structure of C_p, and the facts that (i) we are dealing with an incremental model, and (b) that controls for nonparticipating generators do not vary from the nominal,

$$u[K] = C_p^T u_p[K]. \quad (58)$$

Applying (58) to the dynamical model of (56) gives

$$V_L[K+1] - V_L[K] = C_V K_{inv} K_{mat} C_p^T u_p[K] + (C_V K_{inv} X_{dd'} D_Q - J_{LL}^{-1}) d[K]. \quad (59)$$

Equation (59) expresses the dynamics of pilot voltages throughout the system in terms of load disturbances and voltage controls at participating generators. We will transform this expression with another expression for intersubsystem interactions, our goal being to determine a coordinate the voltage set points at participating generators so that a desired schedule of reactive power/current flows between subsystems is followed.

In Section 4.4, we introduced the concept of a vector interaction variable y in order to describe subsystem dynamics which evolve from outside influences. Denote the interaction variable associated with region i as y_i. From (48), any matrix T_i which satisfies $T_i C^i_V = 0$ renders V^i_L an interaction variable by the linear transformation $y_i[K] = T_i V^i_L[K]$.

Define

$$T = \begin{bmatrix} T_I & 0 & \ldots & 0 \\ 0 & T_{II} & 0 & \ldots \\ \vdots & & \ddots & \\ 0 & \ldots & 0 & T_R \end{bmatrix}, \quad y = \begin{bmatrix} y_I \\ y_{II} \\ \vdots \\ y_R \end{bmatrix} \quad (60)$$

and V_L as in (55). Assume that the T_i are chosen to render the corresponding $V^i_L[K]$ interaction variables. If this is the case, then $y_i[K] = T_i V^i_L[K]$ and, as a consequence,

$$y[K] = T V_L[K]. \quad (61)$$

This implies that (59) can be written in terms of interaction variables by premultiplication by T:

$$y[K+1] - y[K] = T C_V K_{inv} K_{mat} C_p^T u_p[K] - T(C_V K_{inv} X_{dd'} D_Q - J_{LL}^{-1}) d[K]. \quad (62)$$

Recall also an expression for the dynamics of the interaction variables of a subsystem can be found in (49). The interconnected system version can be written as

$$y[K+1] - y[K] = T \hat{J}_{LL}^{-1}(f_L[K] - d[K]), \quad (63)$$

where $f_L[K]$ is a concatenation of the external reactive flows f_i into each subsystem, and $\hat{J}_{LL}$ = block diag J_{LL}, $i = \mathrm{I}, \mathrm{II}, \ldots, R$.

Equation (63) then be substituted into (62), T eliminated, and both sides multiplied by $\hat{J}_{LL}$. This process yields

$$f_L[K] \triangleq F_L[K+1] - F_L[K] = \hat{J}_{LL} C_V K_{inv} K_{mat} C_p^T u_p[K] + (C_V K_{inv} X_{dd'} D_Q + I - \hat{J}_{LL} J_{LL}^{-1}) d[K]. \quad (65)$$

Introduce the proportional control assignment

$$u_p[K] = K_p(V_p[K] - V_p^{opt}), \quad (66)$$

where K_s is a block diagonal matrix consisting of the secondary level K_s, computed for each subsystem, and V_p^{opt} is a given, desired profile for all pilot voltages in the interconnected system. $V_p[K]$ is a setpoint used by the secondary level controllers, a set point which is updated periodically by tertiary-level control. In fact, the

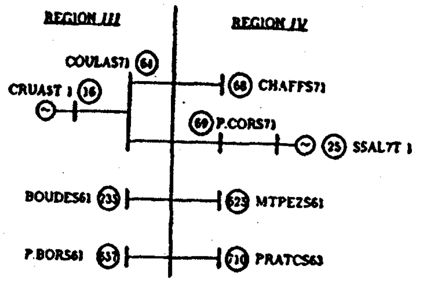


Fig. 6. Tie-lines on the network studied.

taking the change in reactive tie-line flows into consideration.

Over the 3 minute secondary control update interval, however, the initial conditions of the French network (as modeled by CODYSIL) do not provide a sufficiently large change in tie-line flows to demonstrate the effects of ISVC. For this reason, tertiary voltage control is employed at $t = 200$ s to produce this change. Due to reactive flows 3 and 4 (see Table 3), the system does not settle to steady state in 3 minutes: in fact, the settling time is $t = 380$ s, as can be seen from the reactive tie-line flows plotted in Fig. 7(a). When ISVC is applied, the dynamic interactions arising from the flows settle in 3 minutes (Fig. 7(b)), and the system once again has more control reserve in the decentralized regions.

ISVC is also effective when reactive exchange support is created by a large load change. A 1667 MW and 1667 MVAR load drop is implemented in a simulation, and the difference between the case with and without ISVC is obvious (Fig. 8).

6.3. *Tertiary voltage control*

6.3.1. *Basic cases.* A simulation was carried out without tertiary voltage control. As can be seen from Fig. 9(a), pilot node CHAFFS71 does not rise to its setting because generators BUGEYT2 (Fig. 9(b)) and LOIRET3 (Fig. 9(c)) reach their terminal voltage limits. When a tertiary control step is first applied at $t = 200$ s, these generators are pulled away from their constraints (Fig. 10(b,c)), and CHAFFS71 (Fig. 10(a)) reaches its new setting. In addition, generator LOIRET 3 is now producing, instead

Table 3. Aggregate variables

Reactive flow	Region B	Region A
1	P.CORS71	COULAS71
2	CHAFFS71	COULAS71
3	MTPEZS61	BOUDES61
4	PRATCS63	P.BORS61

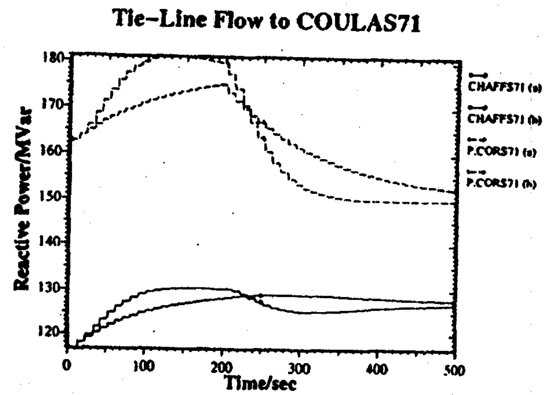


Fig. 7. The effect of improved secondary voltage control (ISVC): (a) without ISVC; (b) with ISVC.

of absorbing, reactive power (Fig. 9(d) and 10(d)).

Important to note is that typically two to three tertiary control steps are necessary to bring the network to a steady state that corresponds to a minimum cost for performance criterion. Figure 11(a) demonstrates that a sequence of tertiary control actions enables the pilot voltages to converge to a system-wide optimum. This optimum is suggested by a particular set of nominal values, and is chosen such that the generator voltages do not exceed their bounds.

Figure 11(b) depicts the result of more simulations involving tertiary voltage control. The control in Fig. 11(b) achieves its voltage regulation with a set of nominal generator voltages lower than those used for the controller in Fig. 11(a). While the generators are pulled away from their constraints, as in the previous case (Fig. 11(a)), the pilot node voltages are generally lower than those of Fig. 11(a). This has implications when the system is expecting different load-change contingencies; for example, a high voltage is desirable when preparing for a load increase, etc.

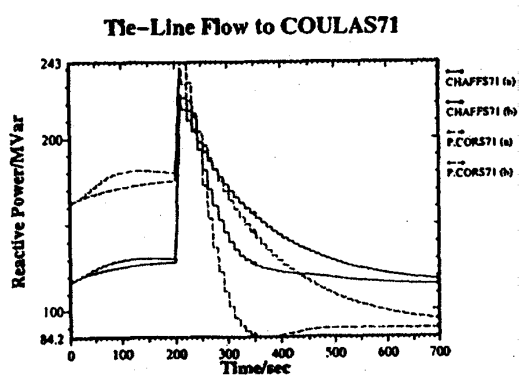
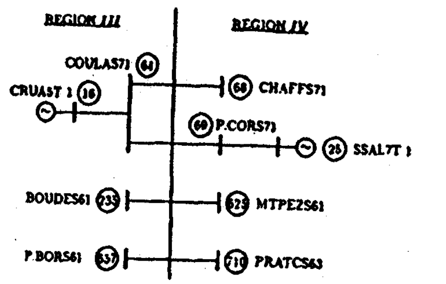


Fig. 8. Improved secondary voltage control (ISVC) at 1667 MW and 1667 MVAR load drop.



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ISVC is also effective when reactive exchange support is created by a large load change. A 1667 MW and 1667 MVAR load drop is implemented in a simulation, and the difference between the case with and without ISVC is obvious (Fig. 8).

6.3. *Tertiary voltage control*

6.3.1. *Basic cases.* A simulation was carried out without tertiary voltage control. As can be seen from Fig. 9(a), pilot node CHAFFS71 does not rise to its setting because generators BUGEYT2 (Fig. 9(b)) and LOIRET3 (Fig. 9(c)) reach their terminal voltage limits. When a tertiary control step is first applied at $t = 200$ s, these generators are pulled away from their constraints (Fig. 10(b,c)), and CHAFFS71 (Fig. 10(a)) reaches its new setting. In addition, generator LOIRET 3 is now producing, instead of absorbing, reactive power (Fig. 9(d) and 10(d)).

Table 3. Aggregate variables

Reactive flow	Region B	Region A
1	P.CORS71	COULAS71
2	CHAFFS71	COULAS71
3	MTPEZS61	BOUDES61
4	PRATCS63	P.BORS61

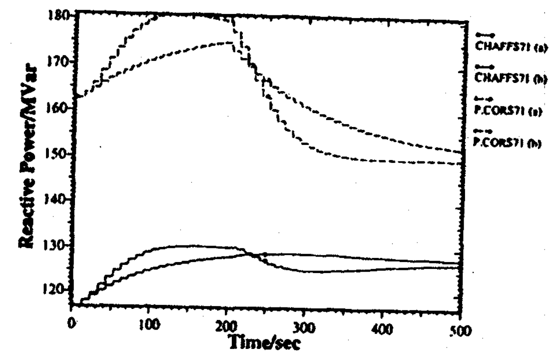


Fig. 7. The effect of improved secondary voltage control (ISVC): (a) without ISVC; (b) with ISVC.

Important to note is that typically two to three tertiary control steps are necessary to bring the network to a steady state that corresponds to a minimum cost for performance criterion. Figure 11(a) demonstrates that a sequence of tertiary control actions enables the pilot voltages to converge to a system-wide optimum. This optimum is suggested by a particular set of nominal values, and is chosen such that the generator voltages do not exceed their bounds.

Figure 11(b) depicts the result of more simulations involving tertiary voltage control. The control in Fig. 11(b) achieves its voltage regulation with a set of nominal generator voltages lower than those used for the controller in Fig. 11(a). While the generators are pulled away from their constraints, as in the previous case (Fig. 11(a)), the pilot node voltages are generally lower than those of Fig. 11(a). This has implications when the system is expecting different load-change contingencies; for example, a high voltage is desirable when preparing for a load increase, etc.

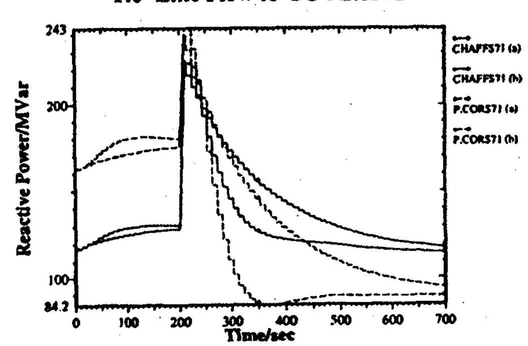


Fig. 8. Improved secondary voltage control (ISVC) at 1667 MW and 1667 MVAR load drop.

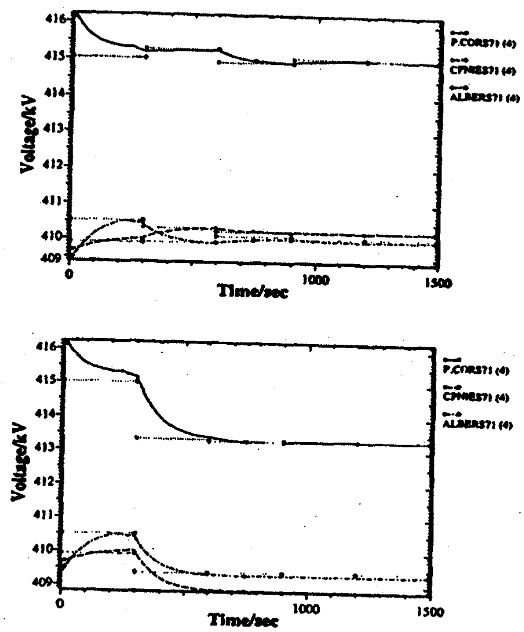


Fig. 11. Pilot voltages with repeated tertiary control: (a) scheduling with higher voltages: (b) scheduling with lower voltages.

large system into regions and an on-line, decentralized, closed-loop, reduced-information structure for controlling regions. For instance, the French power system has been committed to full automation of system-wide voltage regulation, while employing an intuitive reduced information structure at the regional level. The dynamic model of voltage over mid- and long-term horizons, as developed in this paper, is a load-variation and control-driven model, in the sense that any variations in voltages with time are caused only by the control signal or the disturbances. It is shown that for this kind of control-driven model, only a certain number of states can be controlled fully. The maximum number of states that can be controlled fully is equal to the number of controls. Therefore only at most the same number of load voltages as generators can be controlled independently. These states chosen to be independently controlled are referred to as the *pilot load voltages.* The pilot load voltages are controlled within each region by regional controllers, and it is assumed that neighboring regions have negligible effects. In this case, the responsibility for coordinated voltage regulation is shared among regional closed-loop controllers (the secondary voltage controllers) and the operators at the national control center (the tertiary level).

The main goal here is to develop new concepts for coordination of secondary voltage controllers at the national level that preserves a reduced, pilot-point-based information structure. A structurally based decomposition approach is taken to derive the regional voltage dynamic model consisting of local dynamics of individual devices and network coupling.

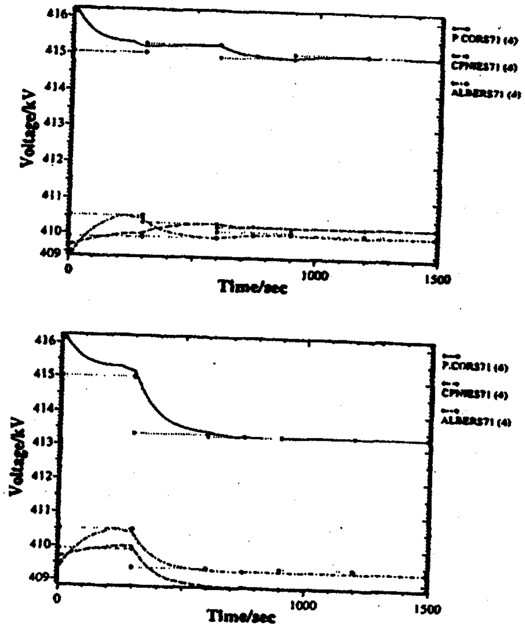
An important breakthrough as a result of structural decomposition is that the interaction variables defined here represent physically measurable variables, such as reactive tie-line flows. This provides a basis for automated feedback control design, because interaction variables are physically measurable.

As the title indicates, the central goals of this paper were to develop models and control strategies relevant for (i) decentralized, regional regulation of load voltages, and (ii) system-wide load voltage coordination, both in response to slow load variations.

The decentralized nature of item (i) is important: it reduces both the communications and scale of computations needed to control load voltages, and reduces the cooperation required from utilities in neighboring regions. In contrast, the concept of coordination in item (ii) involves intrinsic centralization; however, a coordinating function is a necessity when certain regions, while invoking the controls of item (i), approach their control voltage or reactive generation limits. In such a case, by coordinating reactive tie-line flows from adjoining areas, deficient regions can regulate their load voltages while still operating within safe margins.

The principle of vertical separation was introduced to separate fast, primary dynamics from the slower, secondary-level dynamics arising from changing the operational set points of the primary controls. Once this separation was achieved, a linearized, secondary-level model which responds to slow load changes and changing control set points was developed.

With this secondary-level model in hand, control strategies were then discussed. A conventional proportional control was introduced; however, one disadvantage of its use in decentralized applications is that it does not compensate for dynamics induced by tie-line flows from adjacent areas. To address this problem, structural properties of the voltage problem were investigated, and tools developed to overcome the difficulty. A key observation was that if the number of directly regulated (pilot) loads do not exceed the number of generators participating in voltage regulation, voltage dynamics at the regional level are nonsingular. Given this structural nonsingularity, a control strategy which completely compensates for reactive tie-line flows can be developed. Computer simulation proved that the newly



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Next, we addressed the problem of control limits, and their effects on autonomous operation of regions. A real problem is that certain regions may not have the reactive reserves to achieve a desired degree of load voltage regulation on their own. This points out the necessity of a higher level, centralized mechanism by which needy regions can receive assistance from the rest of the interconnected system. We posed inter-regional reactive reserve sharing as problem involving the optimization of a system-wide optimization criterion, and called this higher level function *tertiary control.* An expression which realized tertiary-level rescheduling of secondary control set points was thereupon derived, and then applied to an integrated control strategy. Simulation results were then provided to demonstrate the efficacy of this new, higher level control function.

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