October 14, 2016

Errata

"On the power of (even a little) resource pooling"

J. N. Tsitsiklis and K. Xu

Stochastic Systems, Vol. 2, 2012, pp. 1-66.

This note provides some corrections to the original paper, to be referred to as [TX]. The reason for the correction is the following. The original paper worked with sets of vectors $\mathbf{v}$ that can be obtained from vectors $\mathbf{s}$, through the relation

$$\mathbf{v}_i = \sum_{j=i}^{\infty} \mathbf{s}_j, \qquad i = 0, 1, \ldots \tag{1}$$

However, the set of vectors $\mathbf{v}$ that admit such a representation is not closed, and as a consequence the set $\overline{\mathcal{V}}^M$ defined in Eq. (9) of the original paper is not compact.[1]

It turns out that all of the results in the paper remain valid as long as $\overline{\mathcal{V}}^M$ is redefined in a way that makes it compact. In what follows, we list the necessary changes.

1. Modified Definitions of Certain Sets, on Page 12 of [TX].

Replace the definitions of the sets $\overline{\mathcal{V}}^\infty$ and $\overline{\mathcal{V}}^M$ in Eqs. (9)-(10) of [TX], respectively, by

$$\overline{\mathcal{V}}^\infty = \{\mathbf{v} \in \mathbb{R}_+^\infty : 1 = \mathbf{v}_0 - \mathbf{v}_1 \geq \mathbf{v}_1 - \mathbf{v}_2 \geq \cdots \geq 0\},$$

and

$$\overline{\mathcal{V}}^M = \{\mathbf{v} \in \overline{\mathcal{V}}^\infty : \mathbf{v}_1 \leq M\}.$$

Furthermore, let

$$\overline{\mathcal{V}}_0^\infty = \{\mathbf{v} \in \overline{\mathcal{V}}^\infty : \lim_{i\to\infty} \mathbf{v}_i = 0\}.$$

Wherever the notation $\overline{\mathcal{V}}^\infty$ or $\overline{\mathcal{V}}^M$ is encountered in [TX], it will have the meaning defined in this note, unless a change is indicated in the next section. It turns out that all proofs remain unchanged.

Note that under the weighted L_2 norm defined in Eq. (12) of [TX], the sets $\overline{\mathcal{V}}^M$ are compact, and their union is equal to $\overline{\mathcal{V}}^\infty$. Furthermore, $\overline{\mathcal{V}}_0^\infty$ is the set of vectors in $\overline{\mathcal{V}}^\infty$ that can be represented as in Eq. (1) above, and there is a one-to-one correspondence between elements of $\overline{\mathcal{V}}_0^\infty$ and $\overline{\mathcal{S}}^\infty$, where the latter set is defined in Eq. (8) of [TX]. In particular, any vectors $\mathbf{V}_i^N(t)$ associated with the actual stochastic model automatically belong to $\overline{\mathcal{V}}_0^\infty$.

2. Modifications in the Statements of Certain Results or their Proofs.

1. **Pages 13.** In Definition 1, replace "a function $v(t)$..." with "a continuous function $v(t)$...".
2. **Page 15.** In the second line of the statement of Theorem 2, replace "a state that satisfies…" by "a state in $\overline{\mathcal{V}}^\infty$ that satisfies…".

[1] The authors are grateful to Xiaohan Kang at the Arizona State University for pointing out this error.

3. **Page 19.** Replace the statement of Theorem 5 by the following, which asserts convergence of $\mathbf{s}(t)$, rather than the original stronger statement about convergence of $\mathbf{v}(t)$.
"Given any initial condition $\mathbf{v}^0 \in \overline{\mathcal{V}}_0^\infty$, and with $\mathbf{v}(\mathbf{v}^0, t)$ the unique solution to the fluid model, consider the vector $\mathbf{s}(t)$ associated with $\mathbf{v}(\mathbf{v}^0, t)$. Then,

$$\lim_{t\to\infty} \left\|\mathbf{s}(t) - \mathbf{s}^I\right\|_w = 0,$$

where $\mathbf{s}^I$ is the invariant state of the fluid model given in Theorem 2."

4. **Page 19.** Add the following after the proof of Theorem 5.
"Note. With some additional work, it can be shown that $\mathbf{v}(\mathbf{v}^0, t)$ also converges to $\mathbf{v}^I$, and that this remains true even for initial conditions in $\overline{\mathcal{V}}^\infty$ that are outside $\overline{\mathcal{V}}_0^\infty$."

5. **Page 33.** Replace the first line of the proof of Theorem 2 by the following.
"Let $\mathbf{v}^I$ be an invariant state. If $p = 0$, then $g(\mathbf{v}^I) = 0$ and $\mathbf{v}^I$ obeys a system of linear equations. It is easily verified that if we set the right-hand side of Eq. (15) in [TX] to zero, and use the boundary condition in Eq. (13) of [TX], we obtain a unique solution. Suppose now that $p > 0$. Since $\mathbf{v}_i^I$ is nonincreasing in i, and bounded below by 0, it follows that $\mathbf{v}_i^I - \mathbf{v}_{i+1}^I$ converges to zero. If the sequence $\mathbf{v}_i^I$ does not have finite support, then $g_i(\mathbf{v}^I) = p > 0$ for all i, which implies that for large enough i, the right-hand side of the drift equation (15) in [TX] is nonzero and $\mathbf{v}^I$ is not an invariant state. We conclude that $\mathbf{v}^I$ has finite support and therefore admits a representation of the form (1). Thus, for the rest of the proof, we can work with both $\mathbf{v}^I$ ..."

6. **Page 33, line -4.** Replace "... at all $t > 0$." by "... at all $t > 0$, as long as $\mathbf{v}(0) \in \overline{\mathcal{V}}_0^\infty$.

7. **Page 43, line 1.** Replace "closed and bounded" by "compact".

8. **Page 45, line 14.** Right before "Eq. (105) will imply..." insert: "in light of Theorem 5".

9. **Pages 62-63.** Replace $\overline{\mathcal{V}}^\infty$ by $\overline{\mathcal{V}}_0^\infty$ in the three places: (i) in the first line of Proposition 33; (ii) in the first line of the proof of Proposition 33; (iii) in the second line of p. 63. Furthermore, Eq. (164) of [TX] now follows automatically from the definition of $\overline{\mathcal{V}}_0^\infty$; the sentence that includes Eq. (165) can be removed.