Modern Philosophy and Algebra

Jimmy Rising

Modern philosophy starts with René Descartes. It was his courage and unwavering belief in the principle of doubt which irretrievable shifted the goal, methods, and disposition of philosophy. The work that probably had the greatest effect in driving this shift was Descartes's Meditations.

The Meditations was published in 1641. Although he is best known in philosophy for this work, he is known almost as widely in mathematics for a work he publish just four years earlier: Le Geometrie or the Geometry.

Descartes' thoughts on mathematics and his thoughts on philosophy, although they appear very different, in goals, methods, and dispositions, I claim that they came from a common set of ideas that gave Descartes a unique powerful approach, are silently echoed throughout all of modern philosophy and mathematics, and constitute a fundamental difference between classical and modern philosophies.

1 Synthesis and Analysis

Euclid's and Archimedes's approaches to mathematics extenuate examples. In contrast, Descartes describes a method by which any number of examples can be solved, by translating them into a certain form and translating the solution back out.

The classic mode is the process of synthesis, whereby smaller, complete elements are combined to create more complex elements. For example, Euclid's Proposition 2 of Book I, the method for constructing line of a given length from a point, uses Proposition 1, the method for constructing an equilateral triangle.

Descartes method of analysis does the opposite. In understanding a complex entity, it is taken apart into simpler subparts. This is done in algebra by leaving the exact nature of all of the subparts undefined as variables, and then determining what value they must take.

Note that the context of these method is completely different. Synthesis is fundamentally generative. Euclid does not attempt to solve "problems"– he tries to expand the realm of things that he can "do". The purpose of analysis is entirely for the solving of problems– as Viète, called by many the father of algebra, said, his purpose is "to leave no problem unsolved."

I do not mean to claim that the method of analysis did not exist, or was not in common use during classical times. However, I will argue that modern thought is pervaded through-and-through with these elements, and approaches to mathematics are a reflection of this change.

This shift is central to the difference between ancient and modern philosophy. The idea of "Problems of Philosophy" is prototypically modern, and the reduction of entities that are difficult to grasp to more elemental, concretely, specifically defined pieces is central to modern western philosophy and as it is antithetical to eastern philosophies, I believe it is also foreign or at least misdirected for classical philosophy.

2 Prescriptive and Descriptive

It has been said by some (notably, Adrienne Craig-Williams, a renowned classicist at the Massachusetts Institute of Technology) that modern philosophy is descriptive, in contrast to the prescriptivity of ancient philosophies. Modern philosophers, ever wary of taking an unwarranted step by making too strong a statement, confine themselves to describing how the truth appears, rather than what it suggests, she argues.

Although analysis seems less "gentle" than synthesis, its strength is in the realm of description.

Both analysis and synthesis lead to understanding, but they are different kinds of understanding. The end of taking something apart it to understand it systematically– that is, to understand it as a complex organism of interacting elements.

Synthesis, on the other hand, has a greater potential for clear understanding, as consolation for less directed understanding. The process of synthesis is more a mechanism for understanding the parts than their sum. Given elements, one can synthesize them any number of different ways, and this is particularly clear in Archimedes, where each problem is a reformulation of the same underlying mathematical mechanism. To a modern mathematician, it seem foolish to go through all these trivial examples– why not just approach a problem that one specifically wants to solve? However, this is a misunderstanding, because the point of the examples is not for their own individual analysis, but to understand the mechanism of synthesis that created them.

3 Mathematical Language

There is another reason for this, that is evident in the realm of mathematics. Analysis always involves breaking things down into less imposing elements, and understanding the connections between these elements; that is, analysis is the process of finding the structure of a something. This "structurization" requires its own separate system for describing, and in mathematics, this purpose is served by the symbolic language of mathematics.

At this point, there are at least two distinct "worlds". The first is formed in the human context of the entity being inspected. The second is the mathematical language itself.

Consider the following simple problem: John always saves 10% of his earnings. He has saved

a total of $50. How much money has he earned, total?

This is solved by first describing the problem as an equation:

$.1 \times total = \$50$

Manipulating the equation yields

$total = 10 \times \$50$

This equation is the translation of a potentially very different problem, such as "John gets $10 for every dollar Jane gets. Jane got $10. How much money did John get?" The '10%' of the original has disappeared entirely, translated into a different element. However, any modern student of mathematics would argue the two equations mean the same thing– which is true in the language of mathematics, but not in the world of mathematical symbols.

The use of mathematical formalism is widespread in modern philosophy, and although this formalism takes its form from synthetic math, the use of a "general language" of reason is fundamentally analytical. Both Spinoza and Kant at times use theorems, proofs, definitions, and postulates to force their logic to be more sound. The structure of Leibniz's Monadology falls away from this, but his language is very reminiscent of Euclid's (compare Leibniz first point, "The Monad, of which we will speak here, is nothing else than a simple substance, which goes to make up composites; by simple, we mean without parts." to Euclid's first definition, "A point is that which has no part.").

4 Accurate and Precise

Both analysis and synthesis have strengths and weaknesses as general modes of understanding everything. Each has a sense of hopefulness that the understanding that results is useful.

As the speculative combination of elementary parts, synthesis only results in the particular combinations that are found in the the human world through intuition and cleverness on the part of the thinker. There are similarly endless ways to analyze an entity by speculatively cutting it into pieces, and analysis itself cannot suggest which structures of internal elements will be a useful ways of looking at an entity.

Considering the object of comprehension to be an element for combination in synthesis and the entity for decomposition in analysis, the two modes of thought produce different type of information, when used with less than ideal intuition and cleverness (as is done by human beings). Specifically, with the synthesis of an element with a variety of other elements gives a wide variety of useful knowledge-giving perspectives on the the entity. That is, synthesis, gives accurate information, although it may be only approximate. Analysis, on the other hand, wielding by one who fails to find all of the useful ways to find structure in an entity, can find minutely exact information without consider different perspectives. That is, analysis is powerful in giving precise information.

5 Numerical Understanding

The most general system for understanding truth precisely is numbers and mathematics. Along with the understanding that the whole universe can be described by simple equations and that widely different phenomenon can be made sense of mathematically, comes the belief that everything can be reduced to numbers. Anger is nothing more than patterns of neuron firings and levels of neurotransmitters. Experience is a certain way that atoms dance around each other and interact, which can be described numerically.

The strongest advocate of this view is David Hume, because he came to the conclusion that human reason is just an attempt to find patterns in numbers because it is convenient to humans to do so.

Classical mathematics recognizes a myriad of different kinds of numbers: the evens, the odds, square numbers, the superparticulars, the rationals, the units, and others. Along with the understanding that everything is number comes the understanding that all numbers are interchangeable, through the mathematical language. Classical mathematics did not have the number "zero", because if there was nothing there, then there was nothing there to call nothing. However, the language of mathematics necessitates a way to refer to nothing as a number (like everything else), and zero and negative numbers (and later imaginary numbers) are the result.

Also implicit in mathematical language and analytical thinking is the concept of a variable, which is whatever is left of a structure when all of the other parts are defined. Variables are meaningless as independent entities: their truth comes from the structure of which they form a part. Philosophically, this is the modern philosophers mode of finding the structure that uses an unknown variable. Berkeley knew something about perceptions, and worked everything else into the structure formed by their context, finding all his truth in the "negative space" left by perception.

6 Love of Wisdom and Natural Science

The word "philosophy" means "love of wisdom". Classical philosophers saw their goal as to understand things like virtue, the good, and life.

With the understanding that everything is number came the shifting of this view. No longer can anyone (certainly not number wielding philosophers) answer such questions: the answers are either mathematical in form or meaningless. Philosophy is now "natural science". It occupies the realm of science that instruments have not yet succeeded in gathering sufficient data to develop specific equations.

Spinoza description of emotions shows this very clearly. He does not yet have numerical or symbolic equations for each emotion, but he tries to define them precisely enough that as soon as one can be put in terms of numbers, all of the other equations follow. He defines joy as "a passive state wherein the mind passes to a greater perfection" and then love as "pleasure accompanied by the idea of an external cause". If only everything were so simple!

7 Progress in Philosophy

As philosophy becomes more of a science, it follows the same rules that science follows. Philosophy is a combination of theorems, to be supported and disproved. More importantly, philosophy philosophy is a thing that can achieve or fail to achieve progress.

Progress is also a modern concept. Progress refers to the understanding of the universe as something that evolves. At one time, it had a more primitive structure with more primitive life forms. Then those life forms evolved and adapted and become better, because they were more complex.

Hegel's mechanism for seeing philosophy as a thing that changes and improves over successive conflict and resolution is the epitome of this progress. Philosophical thinkers once viewed the world one way. That philosophers see it differently now is because people between now and then found the faults in past philosophies and made attempts to fix them. In as much as they succeeded, we need not consider the older philosophies. In as much as they failed, there is a conflict and resolution still eminent.

The aspect that this does not take into account is that the method of thought has changed. Progress as conceived by modern philosophers is fundamentally analytical, because it deconstructs the history of philosophy into components which are name "theses" and "antitheses". As long as the mechanism of synthesis continues to be ignored by philosopher today (who

take after the modern philosophers and carry their methods further), truth and wisdom will always be shaded.