Anomalous scaling of mesoscale tropospheric humidity fluctuations

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Abstract. Water vapor fluctuations are measured and analyzed at an unprecedented 10-m resolution throughout the troposphere. Computation of structure functions shows that specific humidity variations observed by research aircraft over the Pacific Ocean exhibit anomalous scaling from about 50 m to 100 km in horizontal range. The scaling laws show different characteristics for the marine boundary layer, the tropical free troposphere, and the extratropical free troposphere. More specifically, boundary-layer humidity fluctuations are less smooth and more stationary than those in the free troposphere, while the extratropical free tropospheric variations are less intermittent than those in the other two regions. The anomalous scaling results argue against passive advection by a spatially smooth flow (chaotic advection) at these scales.

Introduction

As the most effective greenhouse gas in our atmosphere, water vapor has become an increasing focus of research in recent years. However, water vapor is difficult to measure with good resolution, especially in the upper troposphere where it has the most leverage over global radiative forcing. The response time of traditional airborne instruments such as the chilled-mirror device varies with background conditions, and even cryogenically cooled instruments will take up to 20 s to respond. In this paper we report the first scientific results using the full 20-Hz capability of a recently developed laser hygrometer.

The mechanisms producing water vapor variations are of great interest. For example, there has been much theoretical and modeling work done predicated on the idea of chaotic isentropic lateral mixing [e.g., *Emanuel and Pierrehumbert*, 1996]. Our present study provides additional insight into this problem.

Experiment Description

We use data from NASA's Pacific Exploratory Mission in the Tropics, Phase A, conducted in August–September 1996. Its primary mission was to study the impact of human activity on tropospheric chemistry in remote regions over the Pacific Ocean. Two aircraft, a DC-8 and a P-3B, instrumented to measure trace gases, aerosols, and meteorological parameters, made 18 and 21 flights, respectively. For a summary of the experiment see *Hoell et al.* [1999].

The DC-8 carried a laser hygrometer that sampled water vapor at 20-Hz, the best resolution provided by any of the instruments [*Vay et al.*, 1998]. Since the DC-8 air speed varied from about 130 m s^{-1} in the boundary layer to 250 m s^{-1} in the upper troposphere, the corresponding spatial resolution was 6.5–12.5 m, much finer than can be provided by traditional chilled-mirror devices. In our present analysis we use the water vapor data from this instrument exclusively.

Typically each flight was composed of several straight-and-level tracks at different altitudes connected by steep climbs and descents. As the marine boundary layer was a region of interest, each flight usually contained at least one segment at an altitude $\sim$ 300 m or less. For this study we only used the straight-and-level flight segments. We also screened the data carefully for time gaps and periods when the fluctuation level was at the precision limit of the instrument. As a result, we only used a total of 30 data segments from Flights 7 and 9–15. These segments spanned a latitude range of 2°S to 71°S over the Pacific Ocean (see *Hoell et al.* [1999] for flight tracks), with the length of time per segment varying from 11 to 120 minutes. 20 segments were above 3.5 km (free troposphere) and 10 were below 1.5 km (boundary layer). We use the term free troposphere loosely, since at least one of the flight segments we used encountered stratospheric air.

Data Analysis Procedure

The basis for our analysis is the calculation of the structure function as defined by

$$S_q(r) = \langle |\theta(x+r) - \theta(x)|^q \rangle \tag{1}$$

where $\theta(x)$ is a one-dimensional signal (in our case the time series of water vapor mixing ratio), r is some finite difference in x, the brackets denote an ensemble average, and q is the order of the structure function. In turbulence literature S_2 is often called "the" structure function, but we will follow the convention that S_q of any order is a structure function.

We closely follow the procedure outlined by *Davis et al.* [1994] to compute the structure functions. We start with $r = 1$ (corresponding to the sampling interval of 50 ms), then take the average over all possible pairs of the absolute difference moments within the flight segment. This routine is repeated for r increased by a factor of two each time up to $0.05 \times 2^{13} = 409.6$ s. We then invoke Taylor's hypothesis to

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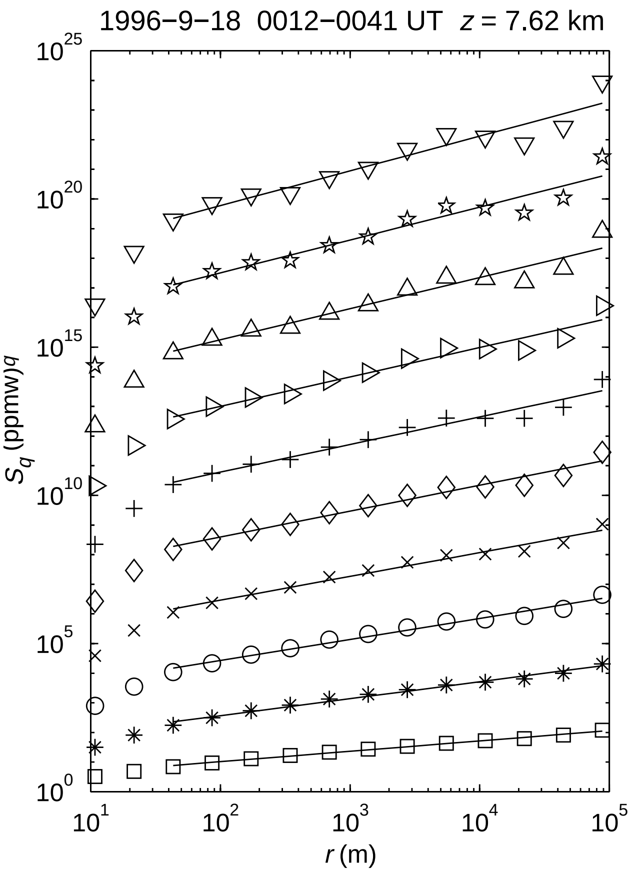

Figure 1. Structure functions calculated up to the 10th order for a 385-km straight-and-level flight segment starting at (10.2°S, 140.3°W) and ending at (12.5°S,142.7°W). The time and altitude are printed at the top of the figure. The order increases in unity steps from bottom to top. Straight lines are fitted to the structure functions from $r = 43$ m to $r = 88$ km in a least-squares sense and plotted.

transform the intervals from time to space using the aircraft speed.

We now ask how the qth-order structure function scales with r. In other words, what is the exponent ζ_q in

$$S_q(r) \propto r^{\zeta_q} \tag{2}$$

If ζ_q/q is a constant, then the relation called ordinary scaling (or monoscaling or monofractal), while a variable ζ_q/q denotes anomalous scaling (or multiscaling or multifractal).

If ζ_q exhibits ordinary scaling, then it does not give us more information about the signal than the Fourier power spectrum, since S_2 is in Fourier duality with the power spectrum for processes with stationary increments. However, if ζ_q shows anomalous scaling, then it will contain information not provided by the power spectrum. For example, both white noise and randomly distributed Dirac delta functions yield a flat power spectrum, even though they are radically different in intermittency. ζ_q contains information that can distinguish between these different types of signals.

Different methods have been proposed to compactly and robustly parameterize the calculated ζ_q curves. *Davis et al.* [1994] advocate the use of ζ_1 to characterize the smoothness of the data along with an intermittency parameter calculated from singular measures. *Pierrehumbert* [1996] introduced a two-parameter fit

$$\zeta_q = \frac{aq}{1 + a\frac{q}{\zeta_\infty}} \tag{3}$$

to the ζ_q curves, where a measures smoothness and ζ_∞ measures multifractality. For convenience we call this the empirical multifractal formula and will use it for our study.

Results

Structure functions were calculated from $q = 1$ to $q = 10$ for each flight segment. A typical free tropospheric example is shown in Figure 1. Note the break in scaling around $r =$ 43 m for all orders. This transition from steeper to shallower slopes with increasing r was seen consistently around the same length scale for free tropospheric segments. We thus calculate the log-log slopes (ζ_q) by fitting a straight line in a least-squares sense between the scaling break and the biggest r. The scaling was generally good beyond the short length-scale break. The corresponding ζ_q values from 1 are plotted in Figure 2. This example departs sharply from monofractal scaling (true for all cases) and is well fit by the empirical multifractal formula of 3 (also generally true).

As we examined the results, we noticed different characteristics emerging for different categories of data. First, the boundary layer results were markedly different from those of the free troposphere. This should come as no surprise given the generally different dynamic and thermodynamic states in these regimes. The Fourier power spectra of trace gas and meteorological variable fluctuations from PEM flights also revealed disparate characteristics for these two altitude regions [*Cho et al.*, 1999]. Second, perhaps not as obvious but still discernible, were differences between tropical (latitudes less than 20°) and extratropical (latitudes greater than 20°) data. For these reasons, we compiled statistics with respect to these three regions.

Table 1 lists the means over the flight segments of the various calculated parameters. (The uncertainty figures are not rigorous error estimates, but simply the standard deviations divided by the square root of N, the number of flight segments.) First, let us look at ζ_1, a parameter that increases with smoothness. For example, $\zeta_1 = 0$ describes a class of signals that includes white noise and Dirac delta functions, while $\zeta_1 = 1$ corresponds to almost everywhere differentiable functions and Heaviside step functions. The smaller ζ_1 implies that boundary-layer humidity variations are less smooth but more stationary than in the free troposphere. Also, $\zeta_1 = 0.25$ is close to the means from two other marine boundary-layer experiments—0.28 from the First ICCP (International Commission on Cloud Physics) Regional Experiment (FIRE) and 0.29 from the Atlantic Stratocumulus Transition Experiment (ASTEX) [*Marshak*

Table 1. Parameter Means

Parameter	Boundary Layer	Free Troposphere Tropical	Free Troposphere Extratropical
N	10	9	11
ζ_1	0.25 ± 0.02	0.37 ± 0.02	0.44 ± 0.02
$\beta = \zeta_2 + 1$	1.46 ± 0.04	1.63 ± 0.05	1.79 ± 0.05
a	0.29 ± 0.03	0.48 ± 0.03	0.49 ± 0.02
ζ_∞	2.7 ± 0.4	2.3 ± 0.5	4.7 ± 0.9

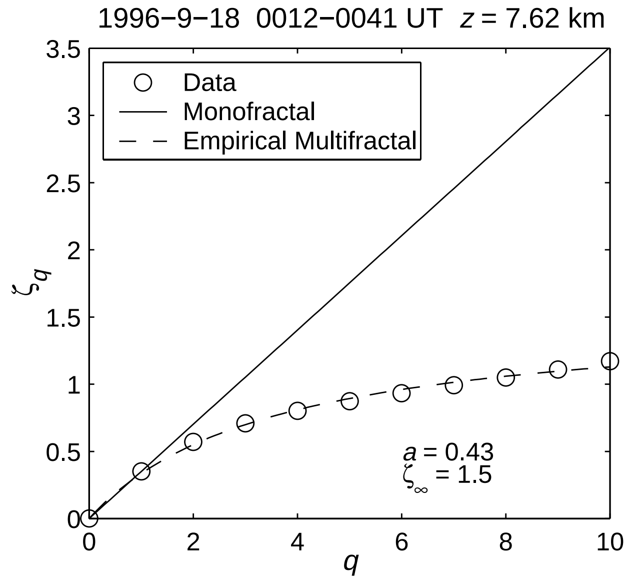

Figure 2. The slopes calculated from the linear fits to the structure functions of Figure 1 are plotted versus order q as circles. The models discussed in the text are also plotted as labeled. The fitted parameters of the empirical multifractal model are shown at the bottom.

et al., 1997]. These experiments measured liquid water content in marine stratocumulus clouds, so the comparison is not direct; however, the scaling range of 20 m–20 km for FIRE and 60 m–60 km for ASTEX were similar to ours. This similarity may reflect a certain universality in the advective dynamics of the marine boundary layer. In contrast, Kolmogorov's classical turbulence theory yields $\zeta_1 \approx 1/3$, which is closest to the value compiled for the tropical free troposphere.

Next, via the Wiener-Khinchine relation that connects the second-order structure function with the Fourier power spectrum, we can obtain the log-log spectral slope, $\beta = \zeta_2 + 1$, where β is the negative exponent of the horizontal wavenumber (k) power spectrum, $E(k) \propto k^{-\beta}$. As an independent check the βs were also calculated from the Fourier power spectra and were found to match these results. All the values lie between 1 and 3, which means that the signal was nonstationary but with stationary increments [*Davis et al.*, 1996]. Again, the boundary-layer value was significantly different from the free tropospheric ones, with the tropical free tropospheric value coming closest to $\beta = 5/3$ of three-dimensional turbulence or quasi-two-dimensional inverse-energy-cascade turbulence.

Of the two fitting parameters of 3, a is distinctly different for boundary layer versus free troposphere. Its behavior for our data set is similar to that of ζ_1, which is not unexpected since $\zeta_1 \approx a$ for $a \ll \zeta_\infty$. The interpretation of a as the most probable continuity exponent [*Pierrehumbert*, 1996] also echoes ζ_1 as a measure of smoothness. On the other hand ζ_∞ measures intermittency, with decreasing values corresponding to higher intermittency. As ζ_∞ goes to infinity, the scaling becomes ordinary (monofractal). The big difference for this parameter is between the extratropical free troposphere and the other regions. Apparently the former has less intermittent water vapor distributions at these scales. Note that our tropical free troposphere means for a and ζ_∞ agree very well with the high cloud variability results of *Pierrehumbert* [1996], which is encouraging since the spatial regime (tropical free troposphere over the Pacific) is the same. However, the length scales studied were much larger, with the scaling interval between 35 and 700 km.

The scaling differences between the three regions can also be illustrated by samples of the raw time series data. Figure 3 shows 200-s samples from, top to bottom, the boundary layer, tropical free troposphere, and extratropical free troposphere. Note the roughness of the boundary layer data and the smoothness and low intermittency of the extratropical free tropospheric data. The (a, ζ_∞) values calculated for the entire flight segments from which the samples were drawn are (0.24, 2.4), (0.43, 1.5), and (0.45, 4.1), respectively.

Summary Discussion

Structure functions of tropospheric humidity fluctuations scaled anomalously between about 50 m and 100 km, the upper limit of our calculations. We found that the two-parameter multifractal formula introduced by *Pierrehumbert* [1996] fit the family of ζ_q curves well in the scaling regime. The distinct differences in the results for three spatial groupings—marine boundary layer, tropical free troposphere, and extratropical free troposphere—showed promise for the use of these two variables in compactly characterizing humidity fluctuation fields, an important step in model parameterization and a possible tool in comparing models with data. Admittedly, given the limited amount of data analyzed, the robustness of the characterization is still to be proven, but we hope to work on this in the future with other data sets.

To briefly summarize the differences in the three regions, the boundary layer fluctuations were less smooth than those in the free troposphere. This difference is likely due to the higher rate of phase changes of water near the surface, which is the source and dominant sink of H_2O in the troposphere, or to convective rolls and turbulence. The more interesting difference was that in intermittency (since it cannot be gleaned from traditional parameters such as the Fourier

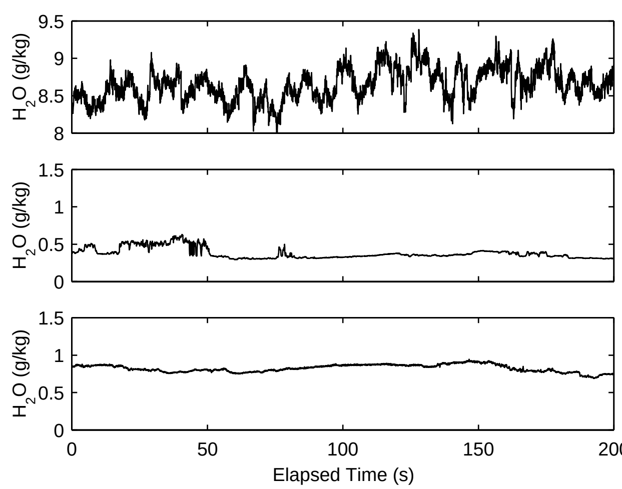

Figure 3. Representative 200-s specific humidity data samples from the boundary layer (top), tropical free troposphere (middle), and extratropical free troposphere (bottom).

power spectral exponent) between the tropical and extratropical free troposphere, with the former having a more intermittent water vapor field than the latter. Perhaps this can be explained by localized convective towers in the tropical region that inject sudden wet plumes along a horizontal path.

Since we know from theory that passive advection by a spatially smooth flow (chaotic advection) should exhibit ordinary scaling [*Chertkov et al.*, 1995], our results do not support this mechanism for the analyzed length scales. Smooth advection should also yield a logarithmic S_2, which is also not supported by our data. However, the width scale of moisture filaments that are signatures of chaotic isentropic mixing models [e.g., *Pierrehumbert*, 1998] (and are also observed as atmospheric rivers [*Newell et al.*, 1992]) are generally larger than the length scales we were able to analyze here. In fact, since Fourier spectral characteristics of winds and temperatures change at scales above mesoscales [e.g., *Nastrom and Gage*, 1985], scaling relations for water vapor fluctuations may also be different at bigger scales. In the future, we plan to scrutinize longer length scales with the measurements of ozone and water vapor by Airbus in-service aircraft (MOZAIC) program, which has much longer continuous flight segments and cover a much larger area of the globe with several years of data, albeit with lower resolution. We will then be able to address additional issues such as parameter robustness and seasonal variations at that time.

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