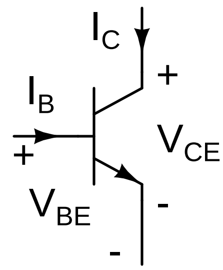
1 All you really need to know about device physics to design bipolar junction transistor circuits...

To design useful transistor circuits, it is important to understand the detailed operation of the bipolar junction device. Ideally, that understanding includes the device physics responsible for the transistor's behavior. Deep knowledge of the physics provides the designer with a solid foundation to understand the circuit models and the trade-offs used in design.



However, it is only necessary to believe the constitutive relationships of the transistor: that the collector current is exponentially dependent on base-emitter voltage

$$I_C = I_S \exp \frac{qV_{BE}}{kT}$$

that the effect of collector voltage can be modeled as a linear term

$$I_C = I_S \exp \frac{qV_{BE}}{kT} \left(1 + \frac{V_{CE}}{V_A}\right)$$

that base current is proportional to collector current (under static conditions)

$$I_B = I_C/\beta_F$$

that collector-base capacitance is related to bias voltage

$$c_\mu = \frac{c_{\mu 0}}{(1 - V_{BC}/\psi_c)^{m_c}}$$

and that diffusion capacitance can be related to transconductance

$$c_b = g_m \tau_F$$

These results can be simply memorized and accepted as the tenets of our faith. From these equations we derive the small-signal and charge-control models that we use in our analysis.

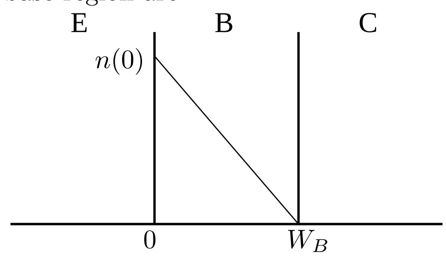
Occasionally, we will wish to delve deeper into the physics behind these relationships to develop deeper insights into device operation. There are five topics from device physics that are most interesting to us as circuit designers: collector current, base current, diffusion capacitance, depletion capacitance, and the temperature dependence of V_{BE}. The following oversimplified derivations provide some background review for those excursions. Additional details can be found in any microelectronics textbook [1, 2, 3].

1.1 Collector Current I_C

In a bipolar transistor, current flows from collector to emitter due to the presence and action of minority carriers in the base region. Majority carriers in the emitter are injected into the base region by the forward-biased base-emitter junction. These injected carriers are minority carriers in the base region, where they flow by diffusion until they are swept into the collector by the reversed-biased base-collector junction.

The carriers (electrons in an npn transistor) flow through the base (from the emitter junction to the collector) by diffusion. To calculate the currents through the transistor, we calculate this diffusion current. The junction voltages control the carrier concentrations. Assuming that the base-emitter junction (at $x = 0$)

is forward biased and that the collector-base junction (at $x = W_B$) is reversed biased, the excess carrier concentrations at the edges of the base region are



$$n(0) = \frac{n_i^2}{N_B} \exp \frac{qV_{BE}}{kT} \quad \text{and} \quad n(W_B) = 0$$

Assuming a linear distribution of carriers in the base, the diffusion current density is

$$J_C = qD_n \frac{dn}{dx} = qD_n \frac{n(0) - n(W_B)}{W_B}$$

thus the collector current is

$$I_C = \frac{qA_E D_n n_i^2}{N_B W_B} \exp \frac{qV_{BE}}{kT} = I_S \exp \frac{qV_{BE}}{kT}$$

This equation is known as the "diode equation" (actually, it is a poor model for diodes, but a very good model for transistors), and it describes the relationship between collector current and base-emitter voltage.

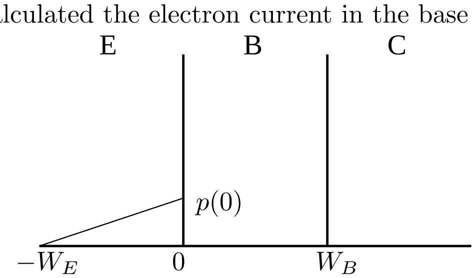
A more careful derivation of the collector current produces the (correct) result

$$I_C = I_S \left(\exp \frac{qV_{BE}}{kT} - 1 \right)$$

so that $I_C = 0$ when $V_{BE} = 0$. However, for all practical current levels, the -1 term is insignificant. In the name of simplicity, it is often ignored.

1.2 Base Current I_B

In an npn transistor, the base current is the reverse injection of holes into the emitter (modern narrow-base devices have negligible recombination in the base). In the transistor, we calculate the hole current in the emitter the same way that we calculated the electron current in the base



$$p(0) = \frac{n_i^2}{N_E} \exp \frac{qV_{BE}}{kT}$$

$$I_B = \frac{qA_E D_p n_i^2}{N_E W_E} \exp \frac{qV_{BE}}{kT}$$

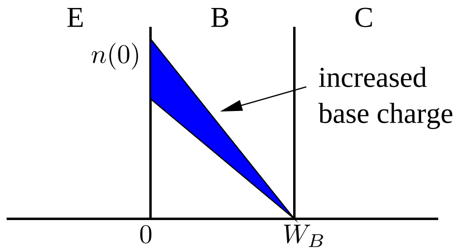
thus the current gain of the transistor is

$$\beta_F = \frac{I_C}{I_B} = \frac{D_n N_E W_E}{D_p N_B W_B}$$

Therefore, to maximize β_F, we want the doping in the emitter to be higher than the base doping, we want the emitter to be wider than the base, and we want the transistor to be npn ($D_n > D_p$).

1.3 Diffusion Capacitance c_b

The diffusion capacitance is due to the distribution of carriers traveling through the base. Increasing the voltage on the base-emitter junction increases the number of carriers in the base. That behavior is a capacitance.



From the area of the minority carrier distribution in the base (a triangle), we can estimate the charge in the base as

$$Q_B = \frac{qn(0)W_B A_E}{2}$$

We can also write the collector current as

$$I_C = qA_E D_n \frac{n(0)}{W_B}$$

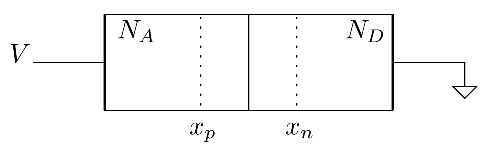
We define the ratio

$$\frac{Q_B}{I_C} = \frac{W_B^2}{2D_n} = \tau_F$$

Thus

$$c_b = \frac{\partial Q_B}{\partial V_{BE}} = \frac{\partial I_C \tau_F}{\partial V_{BE}} = g_m \tau_F$$

1.4 Depletion Capacitance



The capacitance across the depletion region of a pn junction is

$$c = \frac{\epsilon A}{x_d}$$

where x_d is the width of the depletion region. To calculate the width of the depletion region, we start with charge neutrality

$$N_A x_p = N_D x_n$$

and the built-in potential with bias

$$\psi - V = \frac{qN_A x_p^2}{2\epsilon} + \frac{qN_D x_n^2}{2\epsilon}$$

Solving these two equations for the width of the depletion region (a fair amount of algebra)

$$x_d = x_p + x_n = \sqrt{\frac{2\epsilon(N_A + N_D)(\psi - V)}{qN_A N_D}}$$

so the capacitance is

$$c = \frac{\epsilon A}{x_d} = \sqrt{\frac{\epsilon A^2 q N_A N_D}{2(N_A + N_D)(\psi - V)}}$$

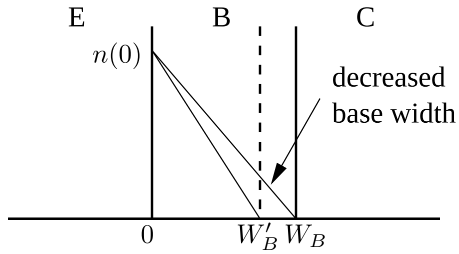
For the collector-base junction of a transistor, this equation is written as

$$c_\mu = \frac{c_{\mu 0}}{(1 - V_{BC}/\psi_c)^{m_c}}$$

where $c_{\mu 0}$ is the zero-bias capacitance, ψ_c is the built-in potential, V_{BC} is the bias across the junction, and m_c is some exponent that describes the doping profile. In the above derivation for an abrupt junction, $m_c = 1/2$. For a linearly graded junction, $m_c = 1/3$.

1.5 The Early Effect: Base-Width Modulation

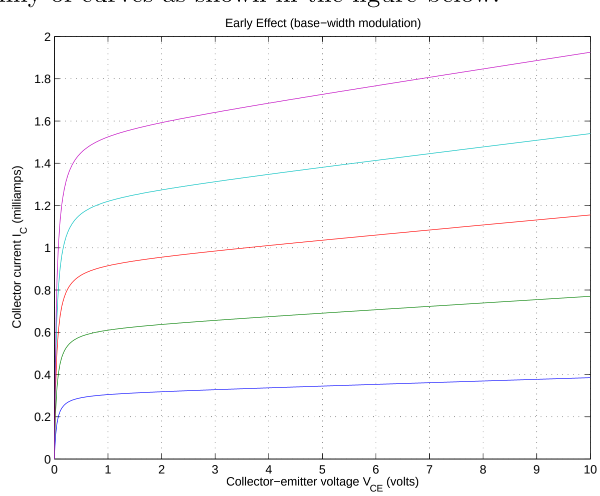
As the collector voltage is increased, the width of the collector-base depletion region increases, as argued above. Thus, the effective width of the base decreases, which increases the slope of the carrier concentration in the base as shown below.



This increase in slope increases the diffusion current through the base, which increases the collector current. This effect is known as "base-width modulation" or the "Early Effect." The dependence of the collector current on collector-base voltage is usually modeled linearly by a term including the Early Voltage, V_A.

$$I_C = I_S \exp \frac{qV_{BE}}{kT} \left(1 + \frac{V_{CE}}{V_A}\right)$$

This equation leads to a family of curves as shown in the figure below.



1.6 V_{BE} Temperature Dependence

We are interested in finding the temperature dependence of V_{BE} for constant I_C [4]. Solving the diode equation for V_{BE}

$$V_{BE} = \frac{kT}{q} \ln \frac{I_C}{I_S}$$

We need to find the temperature dependence of I_S. Starting from the physics, we know that

$$I_S = \frac{qA_E D_n n_i^2}{N_B W_B}$$

The temperature dependence of D_n and n_i^2 can be modeled as

$$D_n = \frac{kT}{q}\mu_n = \frac{kT}{q} C T^{-n}$$

and

$$n_i^2 = D T^3 \exp\left(-\frac{qV_{G0}}{kT}\right)$$

where C and D are some constants that depend on fundamental quantities and processing variables (but not temperature) and V_{G0} is the band gap of silicon extrapolated to absolute zero [5]. Therefore,

$$I_S = \frac{qA_E}{N_B W_B}\left(\frac{kT}{q} C T^{-n}\right) D T^3 \exp\left(-\frac{qV_{G0}}{kT}\right)$$

$$I_S = \frac{T^\gamma}{E} \exp\left(-\frac{qV_{G0}}{kT}\right)$$

where we have collected constants into the "catch-all" E, and $\gamma = 4 - n$. Thus, we can write V_{BE} as

$$V_{BE} = \frac{kT}{q} \ln\left[\frac{I_C E}{T^\gamma} \exp\left(\frac{qV_{G0}}{kT}\right)\right]$$

$$V_{BE} = V_{G0} + \frac{kT}{q} \ln\left[\frac{I_C E}{T^\gamma}\right]$$

Taking the first derivative to find the temperature dependence

$$\left.\frac{\partial V_{BE}}{\partial T}\right|_{I_C} = \frac{k}{q} \ln\left[\frac{I_C E}{T^\gamma}\right] - \frac{kT}{q}\left(\frac{T^\gamma}{I_C E}\right)\frac{\gamma I_C E}{T^{\gamma+1}}$$

Since we can write

$$\frac{k}{q} \ln\left[\frac{I_C E}{T^\gamma}\right] = \frac{V_{BE} - V_{G0}}{T}$$

the temperature dependence can be written as

$$\left.\frac{\partial V_{BE}}{\partial T}\right|_{I_C} = \frac{V_{BE} - V_{G0}}{T} - \frac{\gamma k}{q}$$

which, for typical numbers of $V_{BE} = 600$ mV, $V_{G0} = 1.205$ V, $\gamma = 3.2$, and $T = 300$ K, is

$$\left.\frac{\partial V_{BE}}{\partial T}\right|_{I_C} = -2.01 \text{ mV}/K - 0.28 \text{ mV}/K$$

This result is usually expressed as $\partial V_{BE}/\partial T = -2$ mV/$^\circ C$. In addition, we can calculate the curvature

$$\frac{\partial^2 V_{BE}}{\partial T^2} = -\frac{k}{q}\frac{T^\gamma}{I_C E}\frac{\gamma I_C E}{T^{\gamma+1}} = -\frac{\gamma k}{qT} \approx -1\ \mu\text{V}/K^2$$

1.7 List of Physics Symbols

A_E emitter area

D_n diffusion constant for electrons

D_p diffusion constant for holes

μ_n electron mobility

N_A concentration of acceptor atoms

N_B base doping concentration

N_D concentration of donor atoms

N_E emitter doping concentration

n_i intrinsic carrier concentration

V_{G0} band-gap voltage at absolute zero

W_B base region width

W_B emitter region width

x_n depletion region width on n side

x_p depletion region width on p side

References

[1] Paul E. Gray and Campbell L. Searle. *Electronic Principles: Physics, Models, and Circuits.* Wiley, New York, 1969.

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[4] James K. Roberge. *Operational Amplifiers: Theory and Practice.* Wiley, New York, 1975.

[5] Paul R. Gray and Robert G. Meyer. *Analysis and Design of Analog Integrated Circuits.* Wiley, New York, 1993.

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