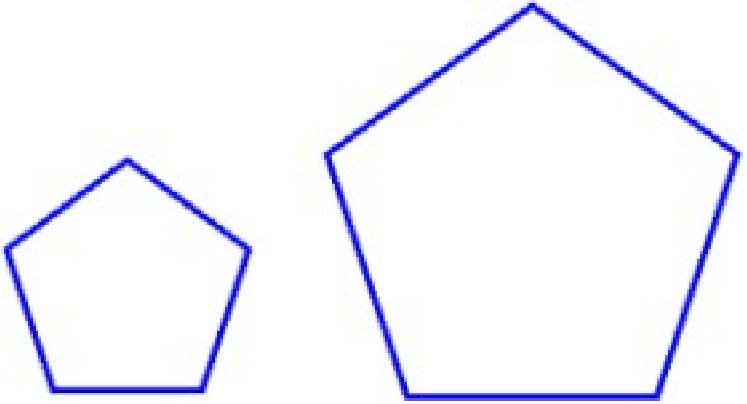
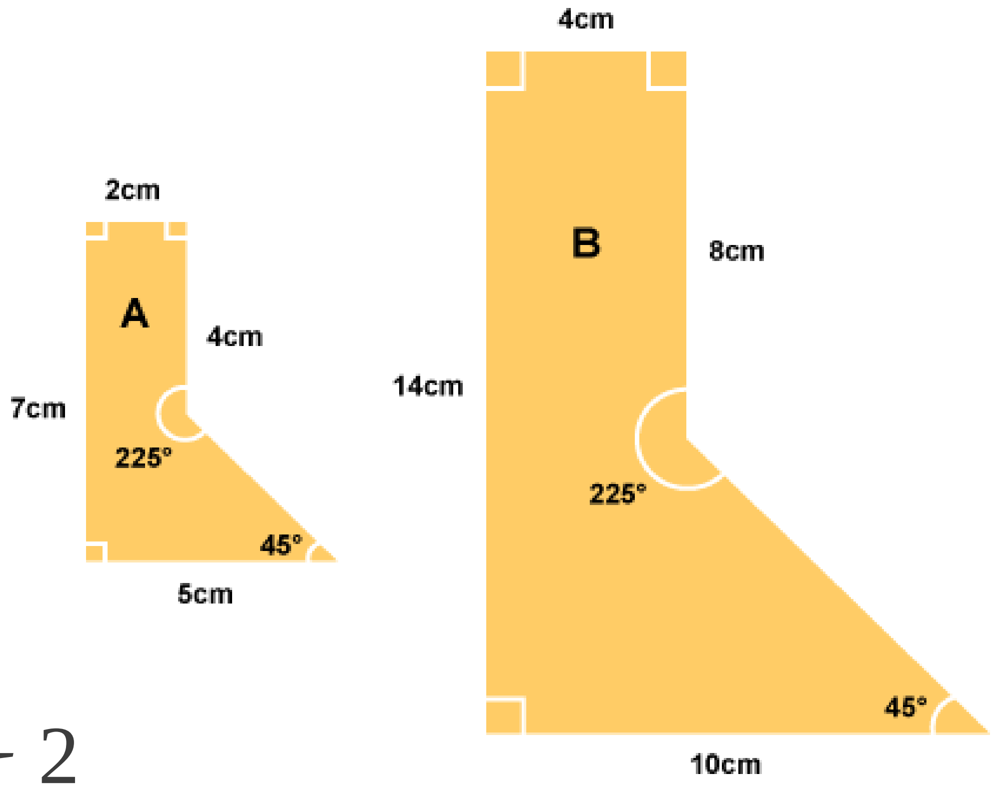
Scaling Anomaly and Atomic Collapse
Collective Energy Propagation at Charge Neutrality

Leonid Levitov (MIT)

Electron Interactions in Graphene
FTPI, University of Minnesota 05/04/2013

Scaling symmetry: a primer



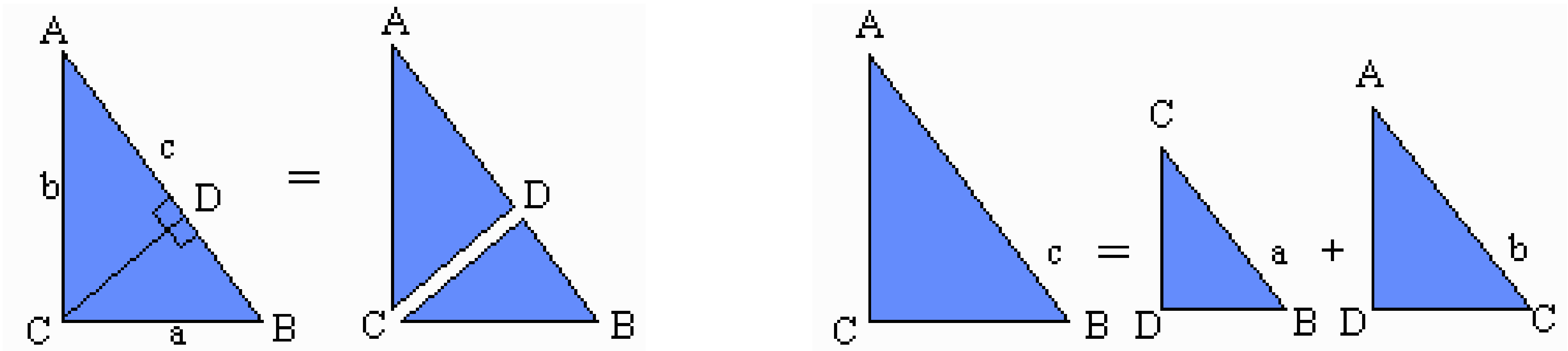


Invariants: angles

Observables: $A \sim L^2$

For similar shapes areas increase as squares of the corresponding sides

Scaling symmetry: a primer



$$s\,c^2 = s\,a^2 + s\,b^2$$

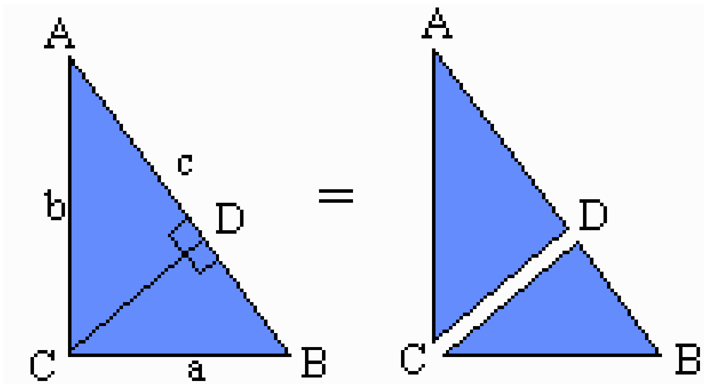
Use scaling to prove Pythagorean theorem

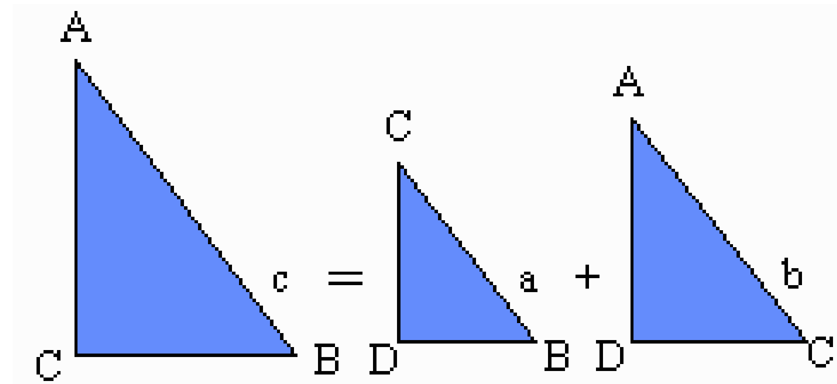
1. Similar triangles

2. Areas add up

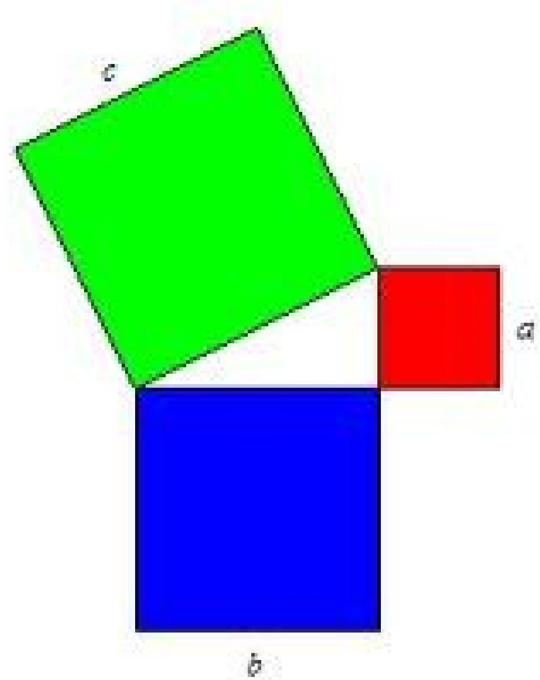
3. (the corresponding sides)^2 add up

Scaling symmetry: a primer





$$c^2 = a^2 + b^2$$



Atomic collapse

Dirac-Kepler problem at large Z, atomic collapse

W. Greiner, B. Muller & J. Rafelski, QED of Strong Fields (Springer, 1985).
I. Pomeranchuk and Y. Smorodinsky, J. Phys. USSR 9, 97 (1945)
Y. B. Zeldovich and V. S. Popov, Usp. Fiz. Nauk 105, 403 (1971)
V. S. Popov, "Critical Charge in Quantum Electrodynamics," in: A. B. Migdal memorial volume, Yadernaya Fizika 64, 421 (2001) [Physics of Atomic Nuclei 64, 367 (2001)]

Coulomb impurity in graphene

K. Nomura & A. H. MacDonald, PRL 96, 256602 (2006).
T. Ando, J. Phys. Soc. Jpn. 75, 074716 (2006).
R. R. Biswas, S. Sachdev, and D. T. Son, PRB76, 205122 (2007)
D. S. Novikov, PRB 76, 245435 (2007)

Atomic collapse in graphene

A. V. Shytov, M. I. Katsnelson, and L. S. Levitov, PRL 99, 236801 (2007)
V. M. Pereira, J. Nilsson, and A. H. Castro Neto, PRL 99, 166802 (2007)
A. V. Shytov, M. I. Katsnelson, and L. S. Levitov, PRL 99, 246802 (2007)
M. M. Fogler, D. S. Novikov, and B. I. Shklovskii, PRB76, 233402 (2007).
I. S. Terekhov, A. I. Milstein, V. N. Kotov, O. P. Sushkov, PRL 100, 076803 (2008)
V. N. Kotov, B. Uchoa, and A. H. Castro Neto, PRB 78, 035119 (2008).

Experiment

Y. Wang ... and M. F. Crommie, Nat. Phys. 8, 653 (2012); Science March 2013
E. Y. Andrei, this conference

The Dirac-Kepler problem

$$H = -i\hbar v \vec{\sigma}\vec{\nabla} - \frac{Ze^2}{r}$$

Scaling symmetry

$$r' = a^{-1} r, \quad \nabla' = a \nabla, \quad H' = aH$$

The Dirac-Kepler problem

$$H = -i\hbar v \vec{\sigma}\vec{\nabla} - \frac{Ze^2}{r}$$

Scaling symmetry

$$r' = a^{-1} r, \quad \nabla' = a\nabla, \quad H' = aH$$

Invariant: $\beta = Ze^2/\hbar v$

The Dirac-Kepler problem

$$H = -i\hbar v \vec{\sigma}\vec{\nabla} - \frac{Ze^2}{r}$$

Scaling symmetry

$$r' = a^{-1} r, \quad \nabla' = a\nabla, \quad H' = aH$$

Invariant: $\beta = Ze^2/\hbar v$

Scaling for observables:

a) scattering phases energy-independent

b) conductivity $\sigma \sim E_F^2 \sim n$

c)

The Dirac-Kepler problem

$$H = -i\hbar v \vec{\sigma}\vec{\nabla} - \frac{Ze^2}{r}$$

Scaling symmetry

$$r' = a^{-1} r, \quad \nabla' = a\nabla, \quad H' = aH$$

Invariant: $\beta = Ze^2/\hbar v$

Scaling for observables:

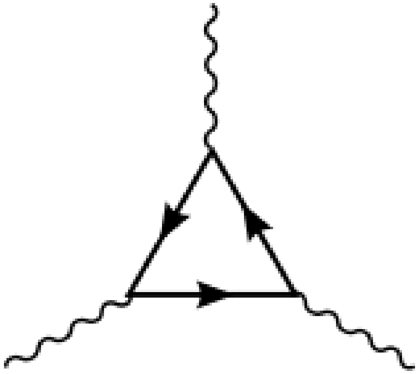
a) scattering phases energy-independent

b) conductivity $\sigma \sim E_F^2 \sim n$

c) **no resonances for any β, no collapse?**

Anomalies in Quantum Physics

Regularized quantum theory can violate a theory's classical symmetry



Ubiquitous: chiral A, conformal A, gauge A...
Scaling A responsible for the large strength of nuclear interactions and quark confinement (asymptotic freedom)

Scaling anomaly:

1. "Global effect" of a short-range cutoff alters the behavior at all energy scales

2. Bound states formation, violation of scaling

3. Essential in theory of quark confinement in QCD developed in 1980's by Gribov (as an alternative to asymptotic freedom)

Scaling anomaly:

1. "Global effect" of a short-range cutoff alters the behavior at all energy scales

2. Bound states formation, violation of scaling

3. Essential in theory of quark confinement in QCD developed in 1980's by Gribov (as an alternative to asymptotic freedom)

Novel hep-condmat connection?

Scaling Anomaly and Atomic Collapse

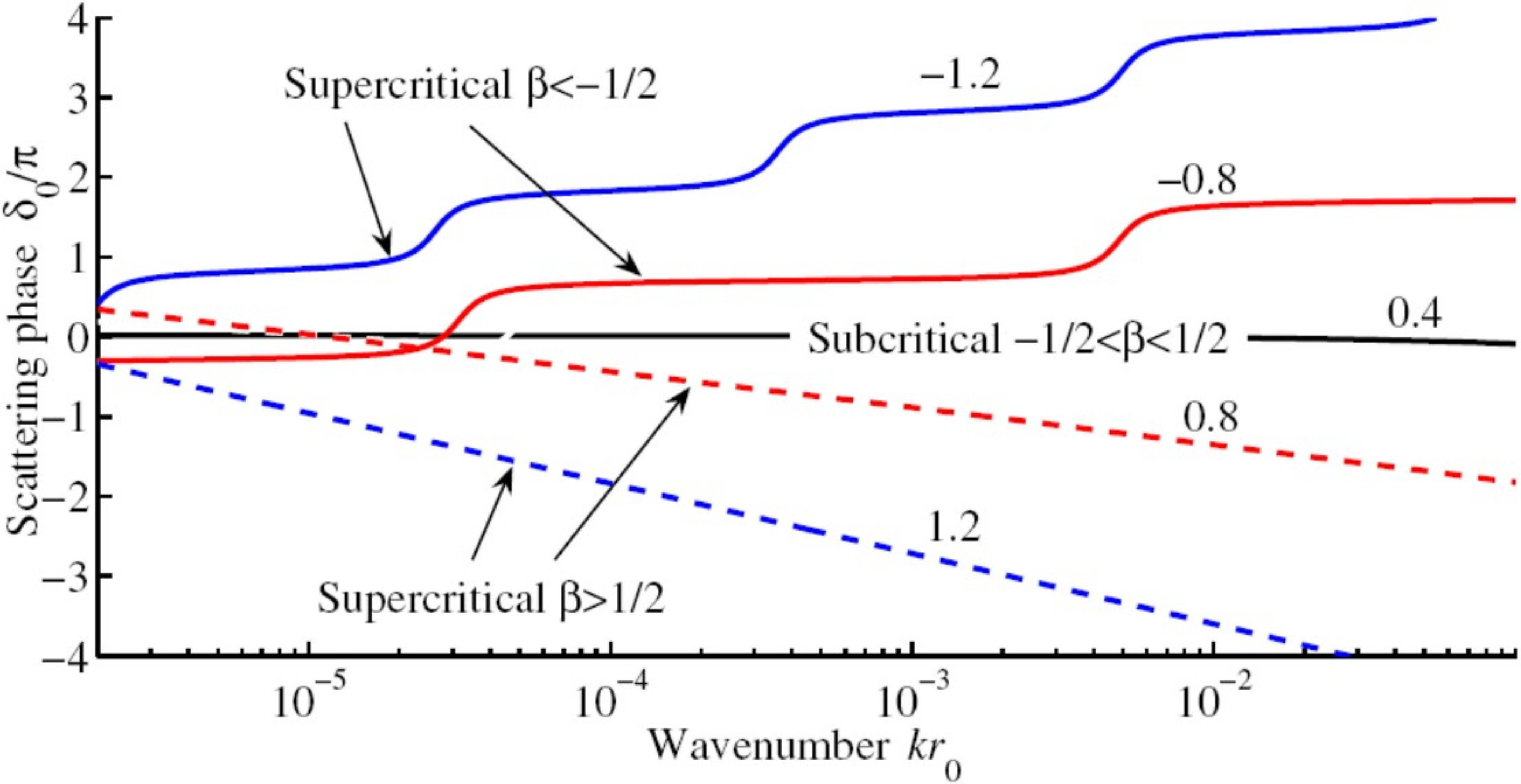
Scaling symmetry for charge impurity (regularized Dirac-Kepler problem):

YES for $|Z|<Z_c$ – regularization insensitive

NO for $|Z|>Z_c$ – for all energies at once!

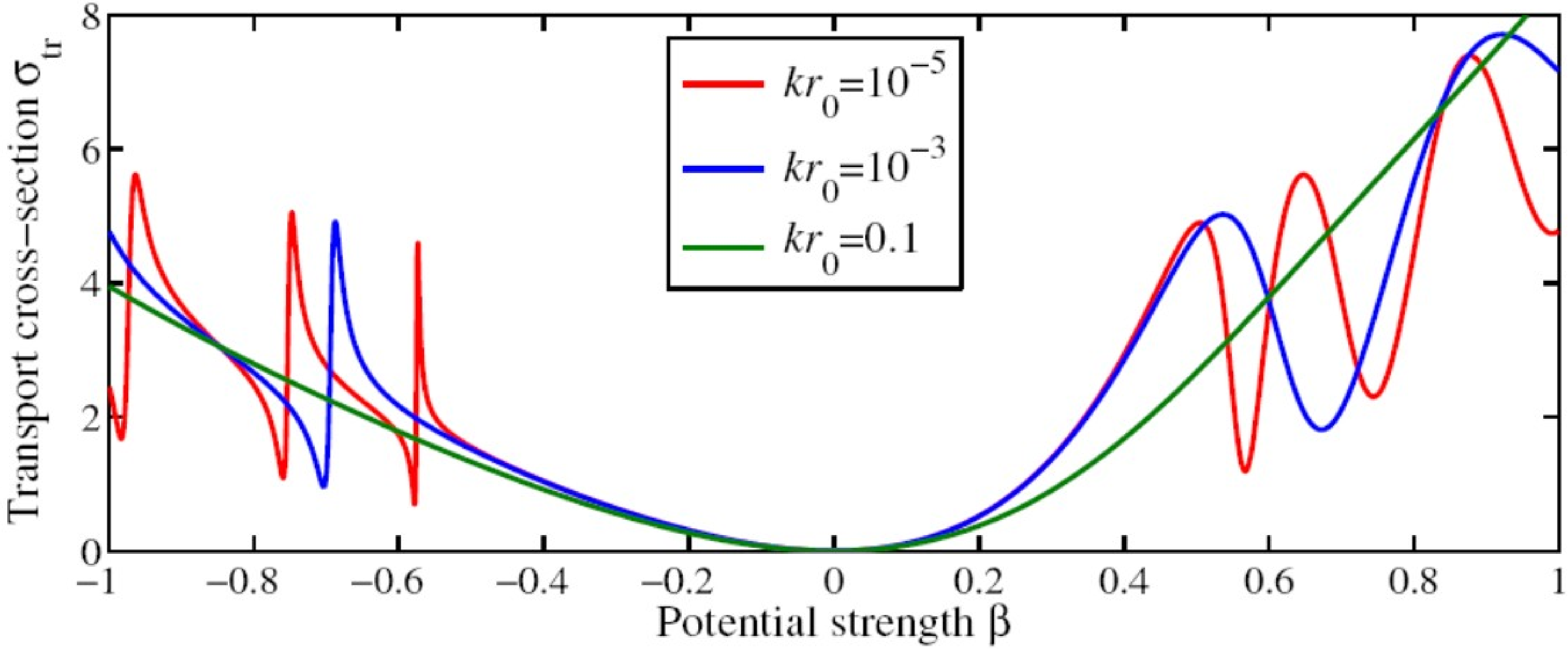
Tunable/switchable anomaly

Scattering phases and resonances



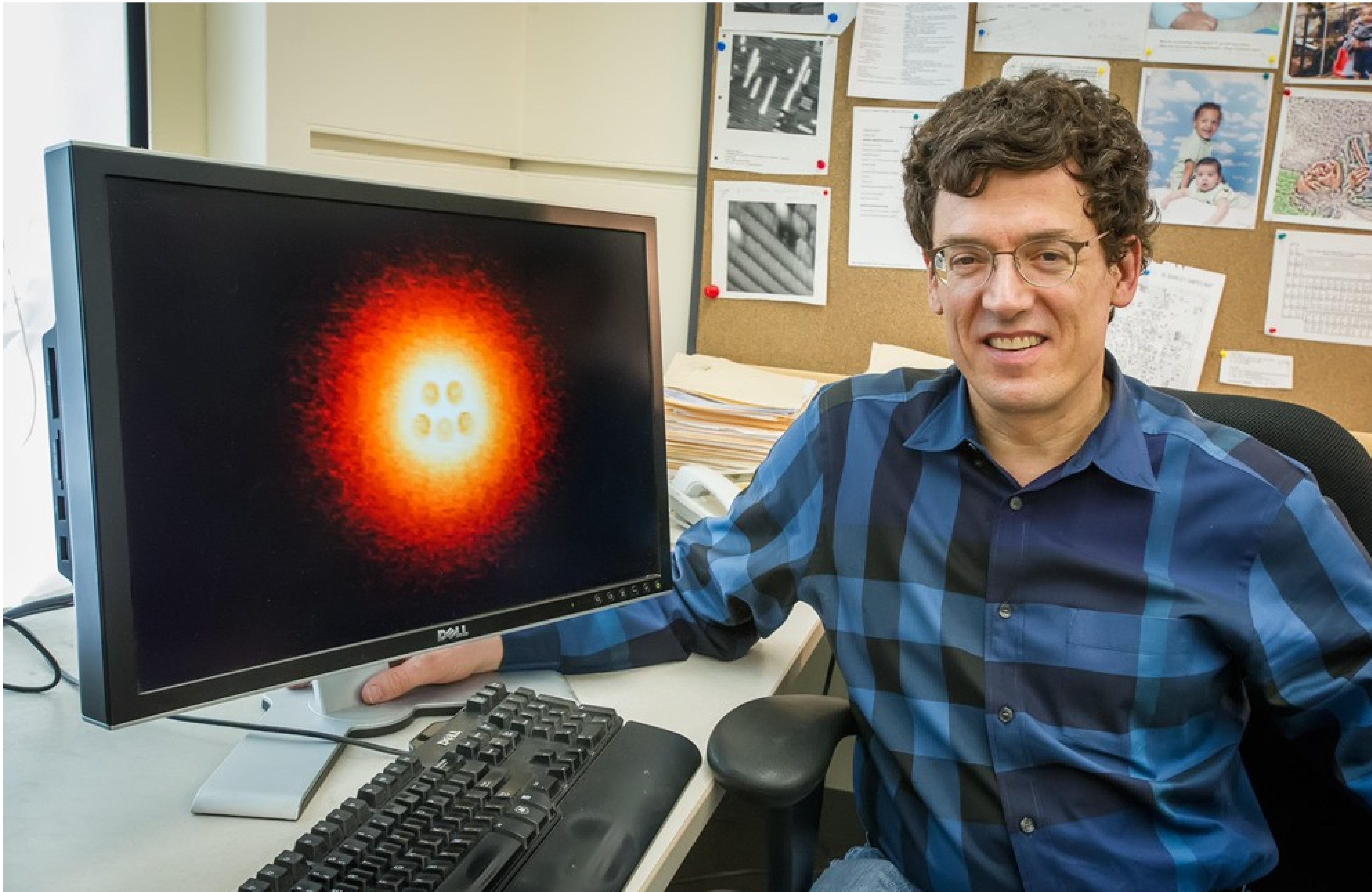
Shytov, Katsnelson and LL, PRL 99, 236801 (2007), PRL 99, 246802 (2007)

Scattering phases (tuning β)



Shytov, Katsnelson and LL, PRL 99, 236801 (2007), PRL 99, 246802 (2007)

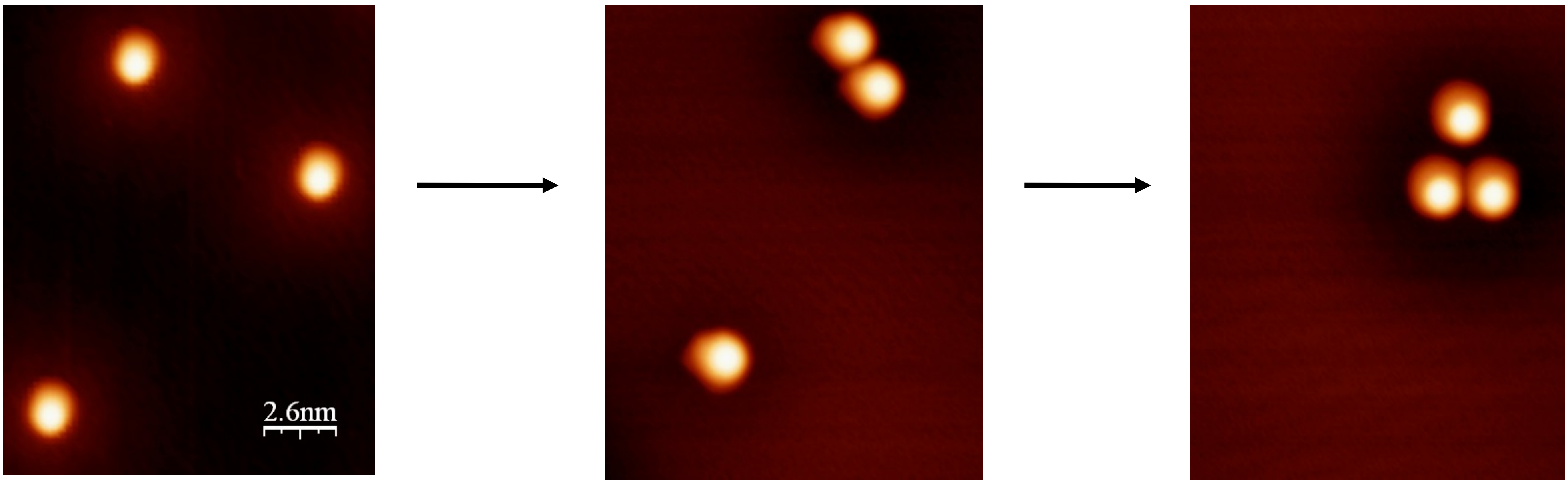
Caught in the act



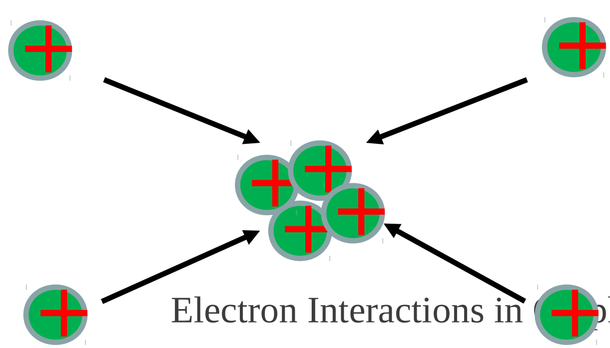
Observation of Atomic Collapse Resonances (Wang et al Science March 7 2013)

Ca Dimers are Moveable Charge Centers

n=3 dimer cluster

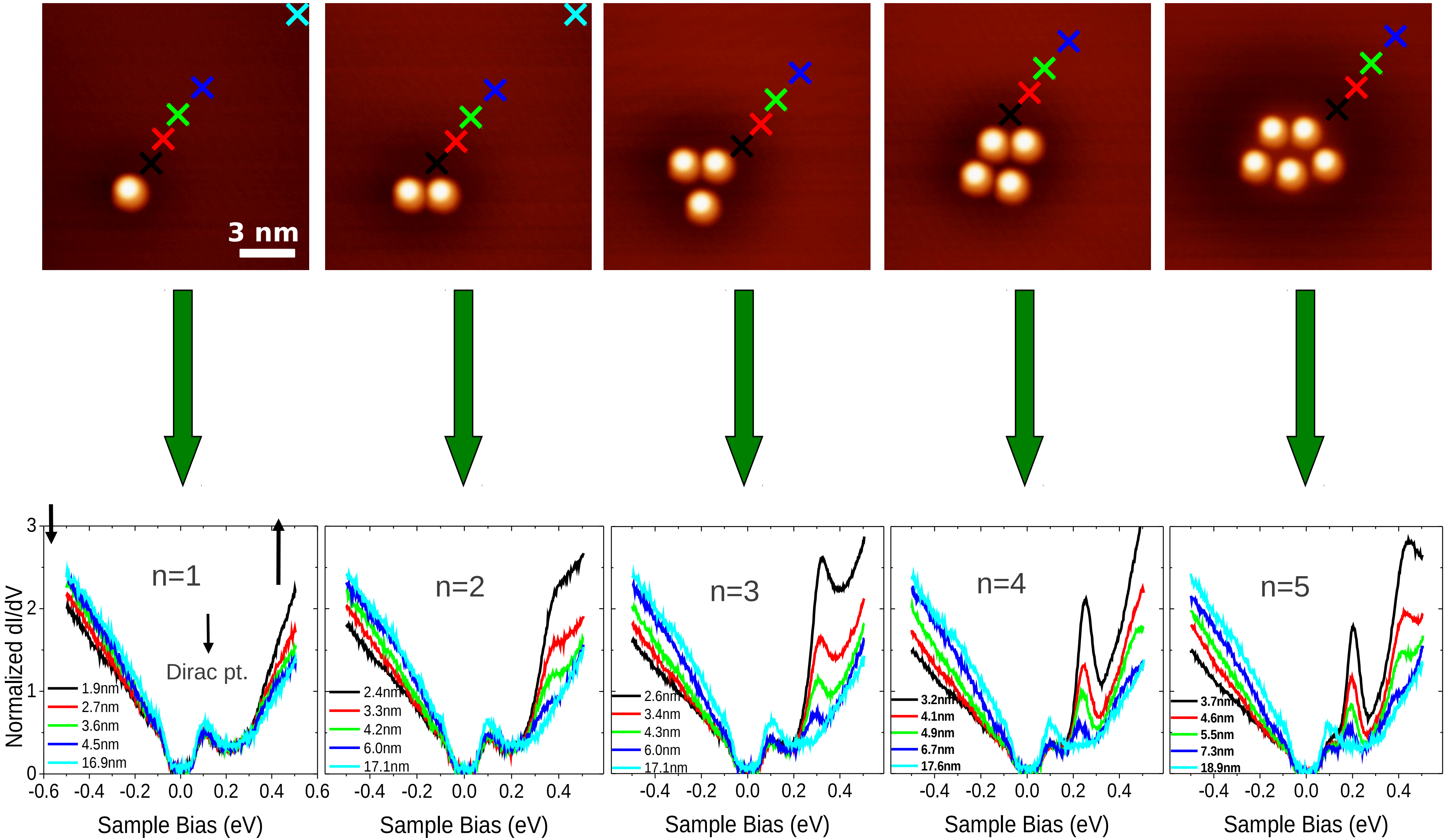


Fabricate Artificial Nuclei

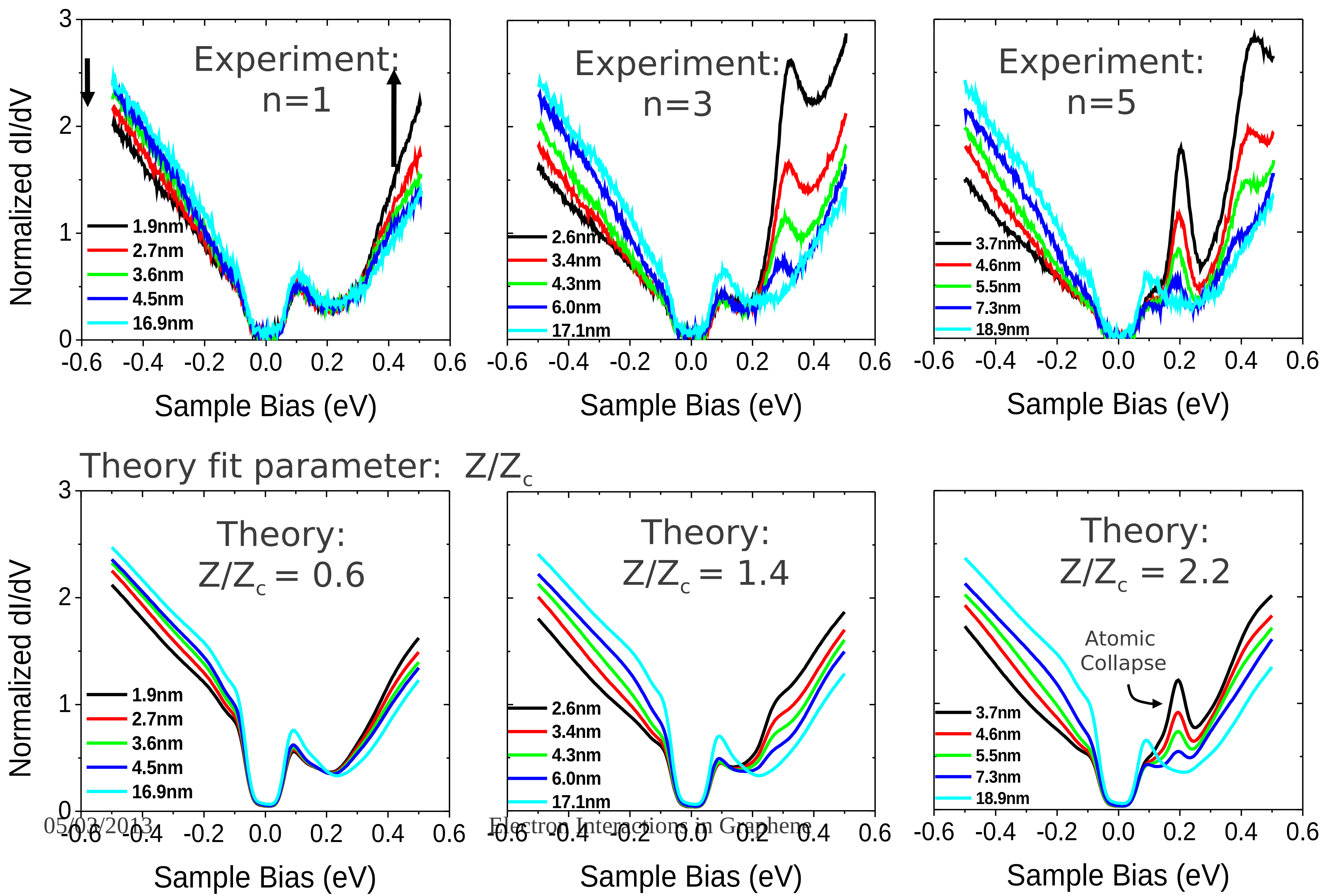


Tune Z above Z_c

Tuning Z by Building Artificial Nuclei from Ca Dimers



Compare to Simulated Behavior



What about scaling symmetry?

Directly probe scaling symmetry (and its violation)?

Adding magnetic field $\vec{A}(r)=\vec{B}\times\vec{r}/2$

$$H=v\vec{\sigma}\left(-i\hbar\vec{\nabla}-e\vec{A}(r)\right)-\frac{Ze^2}{r}$$

Directly probe scaling symmetry (and its violation)?

Adding magnetic field $\vec{A}(r)=\vec{B}\times\vec{r}/2$

$$H=v\vec{\sigma}\left(-i\hbar\vec{\nabla}-e\vec{A}(r)\right)-\frac{Ze^2}{r}$$

sets a length scale and an energy scale:

$$\lambda_B=\left(\hbar/eB\right)^{1/2},\ \ E(B)=v\left(\hbar e B\right)^{1/2}$$

Directly probe scaling symmetry (and its violation)?

Adding magnetic field $\vec{A}(r) = \vec{B} \times \vec{r} / 2$

$$H = v\,\vec{\sigma}\left(-i\hbar\vec{\nabla} - e\vec{A}(r)\right) - \frac{Ze^2}{r}$$

sets a length scale and an energy scale:

$$\lambda_B = (\hbar / eB)^{1/2}, \quad E(B) = v(\hbar e B)^{1/2}$$

Prediction:

all energies must scale with *B* as *sqrt(B)*

Directly probe scaling symmetry (and its violation)?

Adding magnetic field $\vec{A}(r) = \vec{B} \times \vec{r}/2$

$$H = v\vec{\sigma}\left(-i\hbar\vec{\nabla} - e\vec{A}(r)\right) - \frac{Ze^2}{r}$$

sets a length scale and an energy scale:

$$\lambda_B = (\hbar/eB)^{1/2}, \quad E(B) = v(\hbar e B)^{1/2}$$

Prediction:

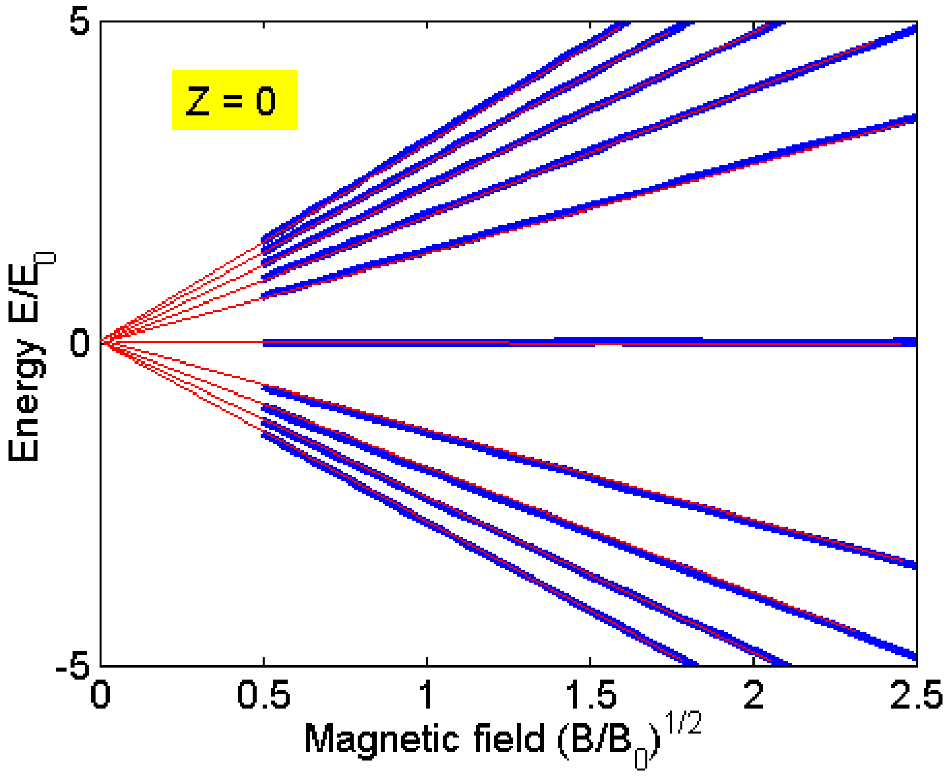
all energies must scale with *B* as *sqrt(B)*

True for both el-imp and el-el interactions

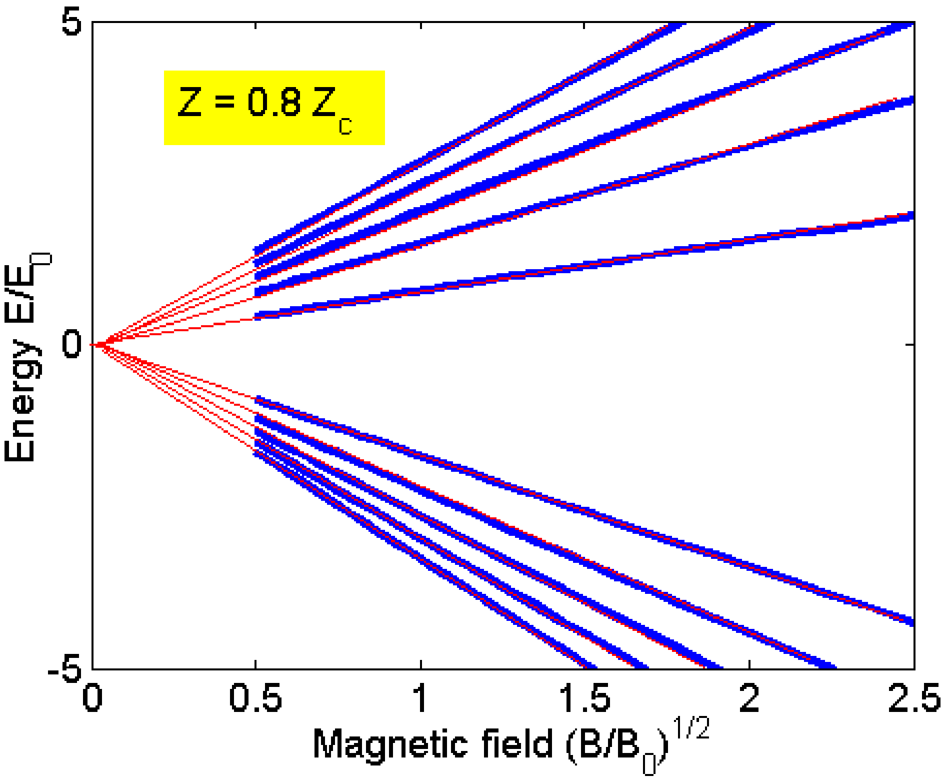
mind running coupling!

Energy spectrum in a B field

Dirac Landau levels (no charge):



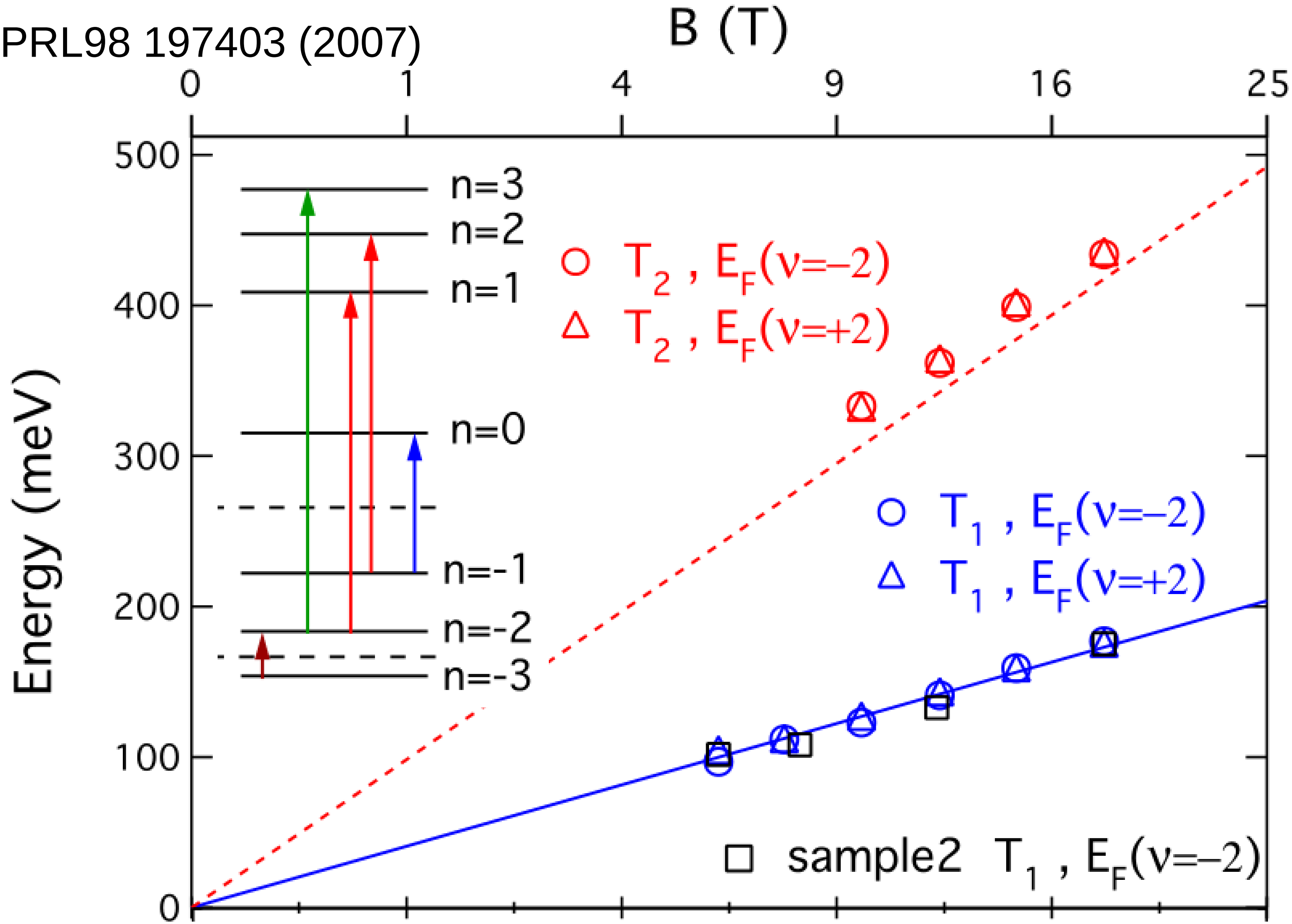
Subcritical charge (DOS near impurity):



all levels show sqrt(B) scaling

Infrared spectroscopy of Landau levels: interactions shifting transitions the sqrt(B) scaling unaffected

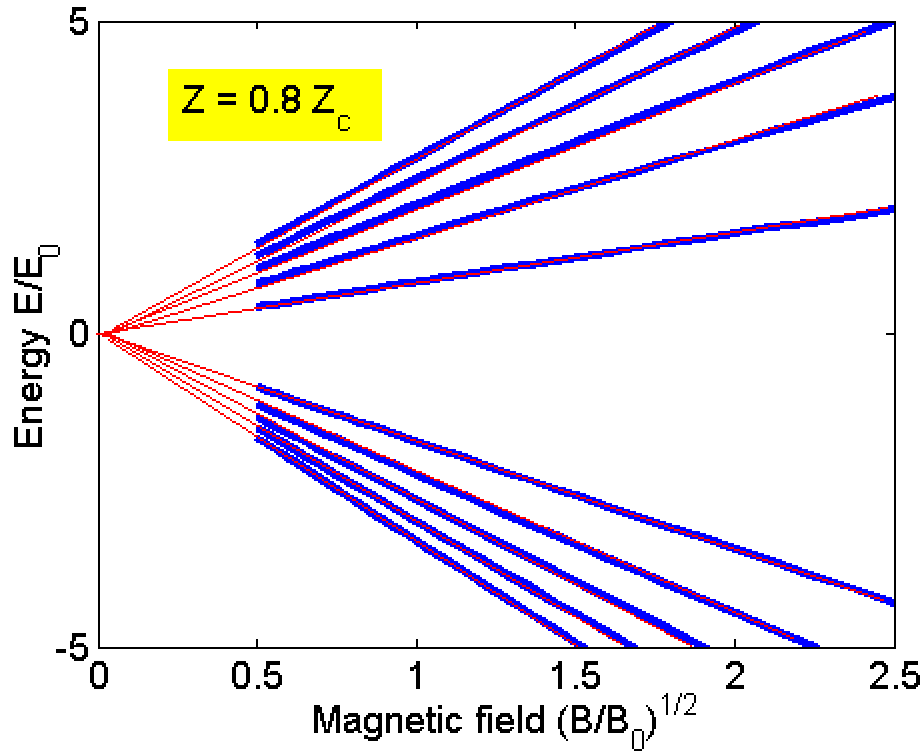
Jiang et al PRL98 197403 (2007)

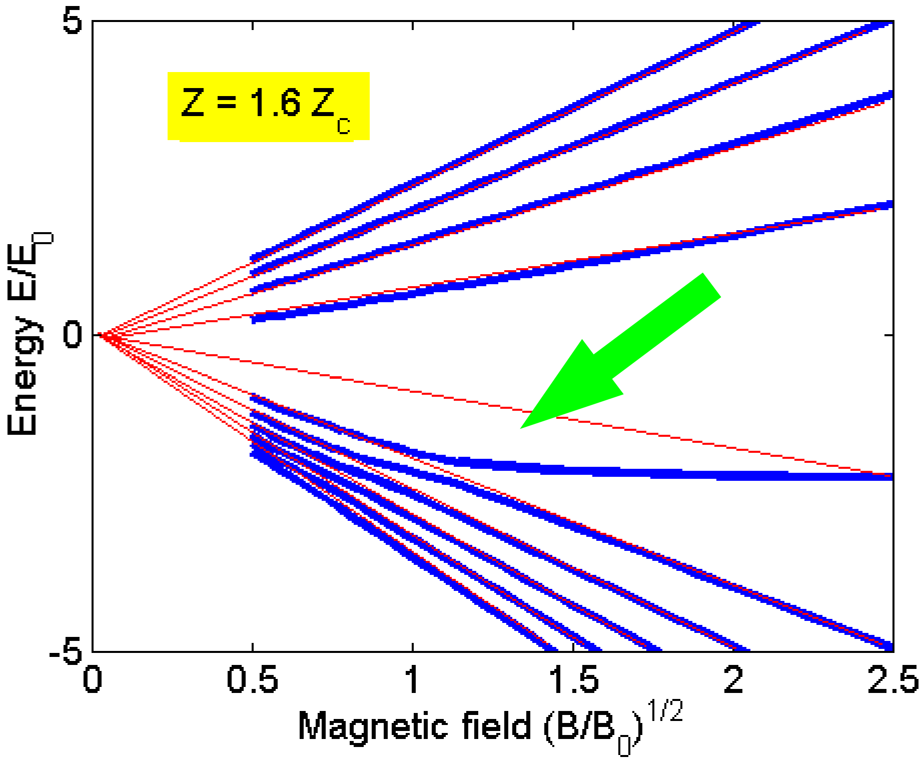


Scaling anomaly

Subcritical charge:

Supercritical charge:





Scaling upheld

Scaling violated

B-field as an experimental knob

1) Scaling symmetry for massless Dirac fermions with 1/r interactions (both el-imp and el-el)

2) Scaling anomaly: scaling violated for supercritical Coulomb potential

3) sqrt(B) scaling breakdown as anomaly litmus test

4) Other knobs (calibration?):
pseudo-B field, strain, curvature, gating

This page intentionally left blank

Collective Energy Propagation at Charge Neutrality

ArXiv:1306.4972 TV Phan, JCW Song & LL

How fast can heat propagate?

Typically, heat propagates through diffusion:

$$\partial_t W = \kappa \nabla^2 T\,, \quad L \sim \sqrt{Dt}$$

How fast can heat propagate?

Typically, heat propagates through diffusion:

$$\partial_t W = \kappa \nabla^2 T, \quad L \sim \sqrt{Dt}$$

In special cases, a compressional wave (second sound)

$$\left(\partial_t^2 - v^2 \nabla^2\right) T = 0$$

reaching speeds as high as
20 m/s (superfluid liquid, He II), 780 m/s (solid, Bi)

How fast can heat propagate?

Typically, heat propagates through diffusion:

$$\partial_t W = \kappa \nabla^2 T, \quad L \sim \sqrt{Dt}$$

In special cases, a compressional wave (second

Is this fast enough?

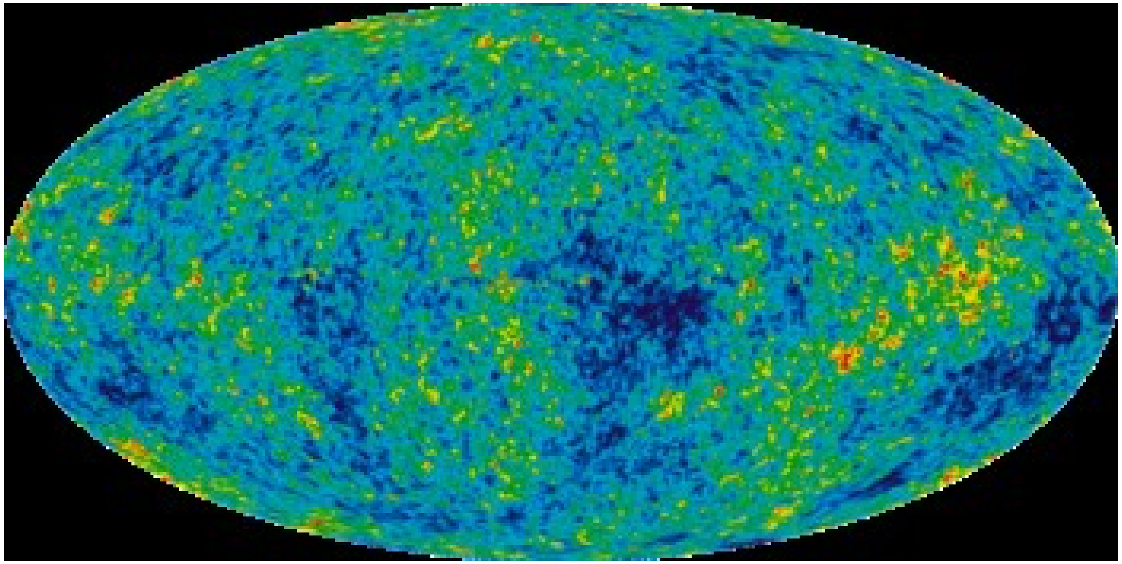
reaching speeds as high as
20 m/s (superfluid liquid, He II), 780 m/s (solid, Bi)

Cosmic sound

Fluid mechanics predicts isentropic waves in a relativistic gas propagating with the velocity

$$c' = c/\sqrt{3}$$

In the early universe “baryon acoustic oscillations” arise due to gravity and radiation pressure. Key in interpreting Cosmic Microwave Background



Energy waves

Relativistic hydrodynamics: energy-momentum conservation (4x4 stress tensor)

$$3\times 3$$

$$\partial_i T_{ij} = 0, \quad T = \begin{pmatrix} E & \vec{j} \\ \vec{j} & \boldsymbol{P} \end{pmatrix}, \quad \boldsymbol{P} = \frac{1}{3} E \delta_{ij}$$

Energy flux and momentum density related by

$$\vec{j} = c^2 \vec{p}$$

giving

$$\left(\partial_t^2 - \frac{1}{3} c^2 \nabla^2\right) E = 0$$

Energy waves

Relativistic hydrodynamics: energy-momentum conservation (4x4 stress tensor)

Cosmic sound in graphene?

Energy flux and momentum density related by

$$\vec{j} = c^2 \vec{p}$$

giving

$$\left(\partial_t^2 - \frac{1}{3} c^2 \nabla^2\right) E = 0$$

Hydrodynamics at CN

- Strongly interacting charge-compensated plasma (Gonzales, Guinea, Vozmediano, Son, Sheehy & Schmalian, Vafek)
- E & P conserved in carrier-carrier collisions
- Rapid exchange of E and P among colliding particles
- Relatively slow P relaxation, very slow E relaxation
- “Inertial mass”: E-flux proportional to P-density

Hydrodynamics at CN

- Strongly interacting charge-compensated plasma (Gonzales, Guinea, Vozmediano, Son, Sheehy & Schmalian, Vafek)
- E & P conserved in carrier-carrier collisions
- Rapid exchange of E and P among colliding particles
- Relatively slow P relaxation, very slow E relaxation
- “Inertial mass”: E-flux proportional to P-density $\vec{j}_w = v^2 \vec{p}$

$$\partial_t W + \vec{\nabla} \vec{j}_W = 0, \quad \partial_t p_i + \nabla_i \sigma_{ij} = 0, \quad \sigma_{ij} = \frac{1}{2} W \delta_{ij}$$

Hydrodynamics at CN

- Strongly interacting charge-compensated plasma (Gonzales, Guinea, Vozmediano, Son, Sheehy & Schmalian, Vafek)
- E & P conserved in carrier-carrier collisions
- Rapid exchange of E and P among colliding particles
- Relatively slow P relaxation, very slow E relaxation
- "Inertial mass": E-flux proportional to P-density $\vec{j}_w = v^2 \vec{p}$

$$\partial_t W + \vec{\nabla}\, \vec{j}_W = 0, \quad \partial_t p_i + \nabla_i \sigma_{ij} = 0, \quad \sigma_{ij} = \frac{1}{2} W \delta_{ij}$$

$$\left(\partial_t^2 - \frac{1}{2} v^2 \nabla^2\right) W = 0, \quad v' = \frac{v}{\sqrt{2}} \approx 0.71 \cdot 10^6\, m/s$$

Hydrodynamics at CN

- Strongly interacting charge-compensated plasma (Gonzales, Guinea, Vozmediano, Son, Sheehy & Schmalian, Vafek)
- E & P conserved in carrier-carrier collisions

Electronic thermal waves x1000 faster than phonon 2nd sound, however x250 slower than cosmic sound

$$\partial_t W + \vec{\nabla}\,\vec{j}_W = 0,\quad \partial_t p_i + \nabla_i \sigma_{ij} = 0,\quad \sigma_{ij} = \frac{1}{2} W \delta_{ij}$$

$$\left(\partial_t^2 - \frac{1}{2} v^2 \nabla^2\right) W = 0,\quad v' = \frac{v}{\sqrt{2}} \approx 0.71 \cdot 10^6\, m/s$$

Compare to plasmons

1. propagating charge density wave
2. requires finite doping
3. frequencies $\omega < E_F$
4. damping: momentum relaxation

$$\partial_t \vec{p} = \rho \vec{E}, \quad \vec{E} = -\vec{\nabla}_r \int V(r - r')\rho(r') d^3 r'$$

$$\partial_t \rho + \vec{\nabla} \vec{j} = 0, \quad \vec{j} = \frac{e}{\tilde{m}} n_0 \vec{p}$$

$$\omega(\omega + i\gamma_p) = \frac{2e^2 E_F}{\tilde{\epsilon} \hbar^2} k$$

Compare to plasmons

1. propagating charge density wave
2. requires finite doping
3. frequencies $\omega < E_F$
4. damping: momentum relaxation

$$\partial_t \vec{p} = \rho \vec{E}, \quad \vec{E} = -\vec{\nabla}_r \int V(r - r')\rho(r')\, d^3 r'$$

$$\partial_t \rho + \vec{\nabla}\, \vec{j} = 0, \quad \vec{j} = \frac{e}{\tilde{m}} n_0 \vec{p}$$

$$\omega(\omega + i\gamma_p) = \frac{2e^2 E_F}{\tilde{\epsilon}\hbar^2} k$$

Compare to plasmons

1. propagating **temperature** wave
2. requires **zero** doping
3. frequencies **$\omega \ll \alpha$^2 kT**
4. damping: momentum relaxation

also Umklapp scattering and viscosity (small)

$$\left(\partial_t - D\nabla^2\right)W + v^2\vec{\nabla}\,\vec{p} = 0, \quad D = \kappa / C$$

$$\left(\partial_t - \nu\nabla^2 + \gamma_p\right)\vec{p} + \frac{1}{2}\nabla W = 0$$

$$\omega\left(\omega + i\gamma_p\right) = \frac{2e^2 E_F}{\epsilon\hbar^2}k$$

$$\left(\omega + iDk^2\right)\left(\omega + i\gamma_p + i\nu k^2\right) = \frac{v^2}{2}k$$

Validity and frequency range

carrier-carrier scattering time $\tau_\epsilon \approx \frac{\hbar}{\alpha^2 kT}$

Kashuba (2008), Fritz et al (2008)

For T=100K find $\tau_\epsilon \approx 60\, fs$

Validity and frequency range

carrier-carrier scattering time $\tau_\epsilon \approx \frac{\hbar}{\alpha^2 kT}$

Kashuba (2008), Fritz et al (2008)

For T=100K find $\tau_\epsilon \approx 60\, fs$

Hydrodynamics valid for $\omega \ll 1/\tau_\epsilon$

$$\left(\omega + iDk^2\right)\left(\omega + i\gamma_p + i\nu k^2\right) = \frac{v^2}{2} k^2$$

Validity and frequency range

carrier-carrier scattering time $\tau_\epsilon \approx \frac{\hbar}{\alpha^2 kT}$

Kashuba (2008), Fritz et al (2008)

For T=100K find $\tau_\epsilon \approx 60\, fs$

Hydrodynamics valid for $\omega \ll 1/\tau_\epsilon$

$$(\omega + iDk^2)(\omega + i\gamma_p + i\nu k^2) = \frac{v^2}{2} k^2$$

Weak damping, ω''<<ω', for $\gamma_p \ll \omega \ll \tau_\epsilon^{-1}$

Few-μm mean free paths (high doping, clean G/BN systems)

sqrt(n) dependence extrapolated to DP yields $\gamma_p^{-1} \approx 0.5\, ps$

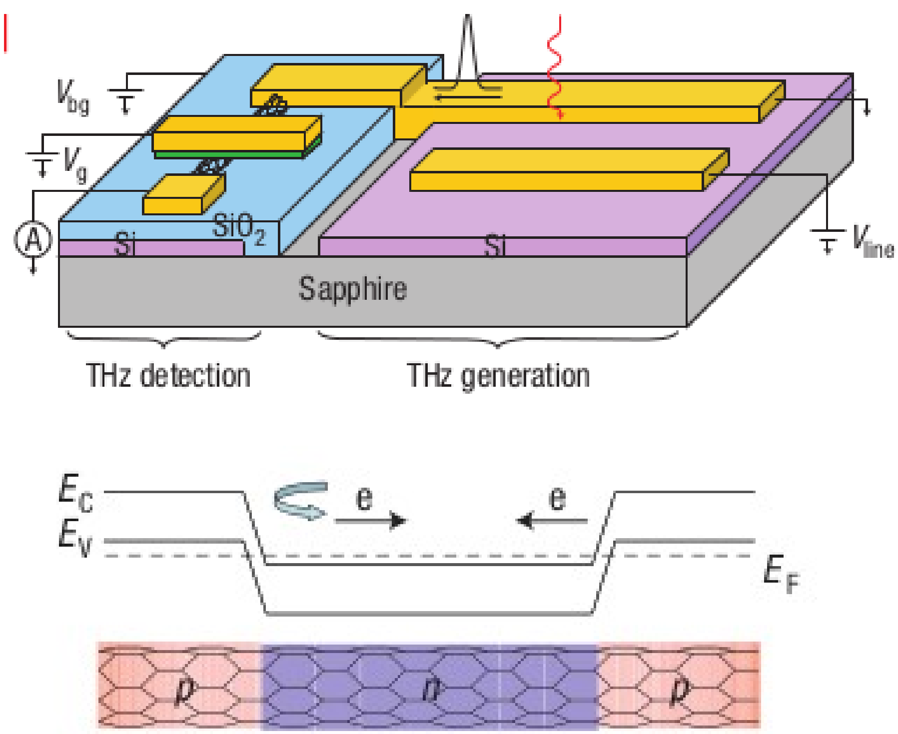
$$0.2\, THz < f < 3\, THz \qquad 0.3\, \mu m < \lambda < 5\, \mu m$$

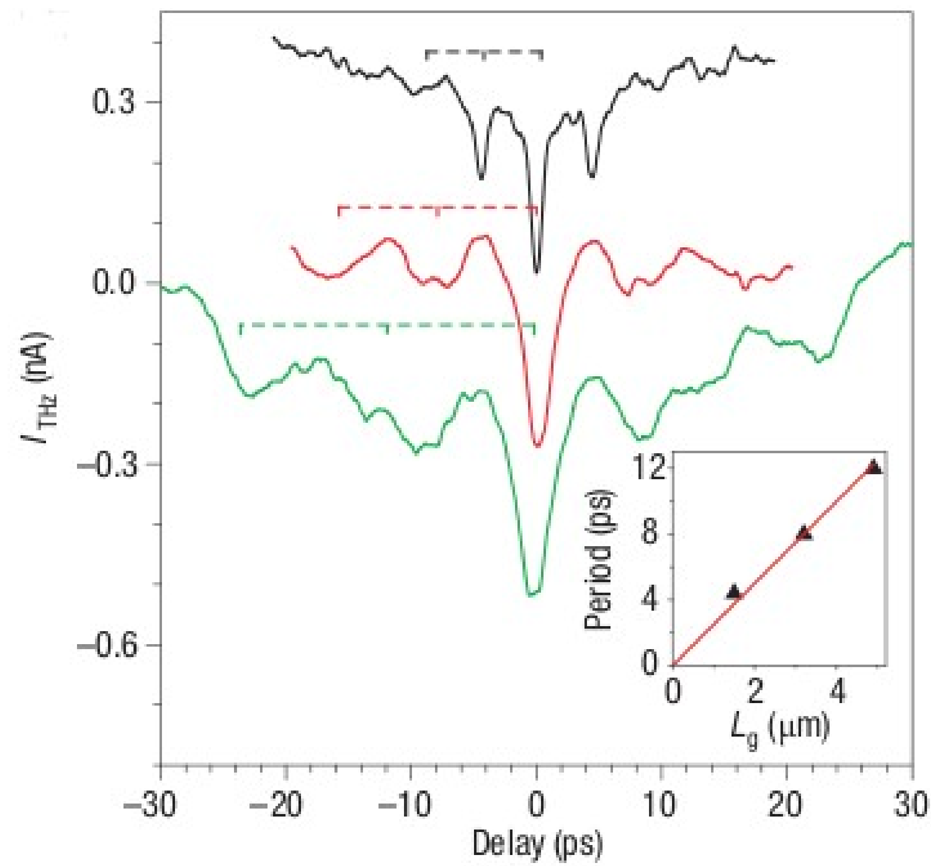
How to observe?

Terahertz time-domain measurement of ballistic electron resonance in a single-walled carbon nanotube

Nature Nano 2008

ZHAOHUI ZHONG[1], NATHANIEL M. GABOR[1,2], JAY E. SHARPING[3†], ALEXANDER L. GAETA[1,3] AND PAUL L. McEUEN[1,2]*



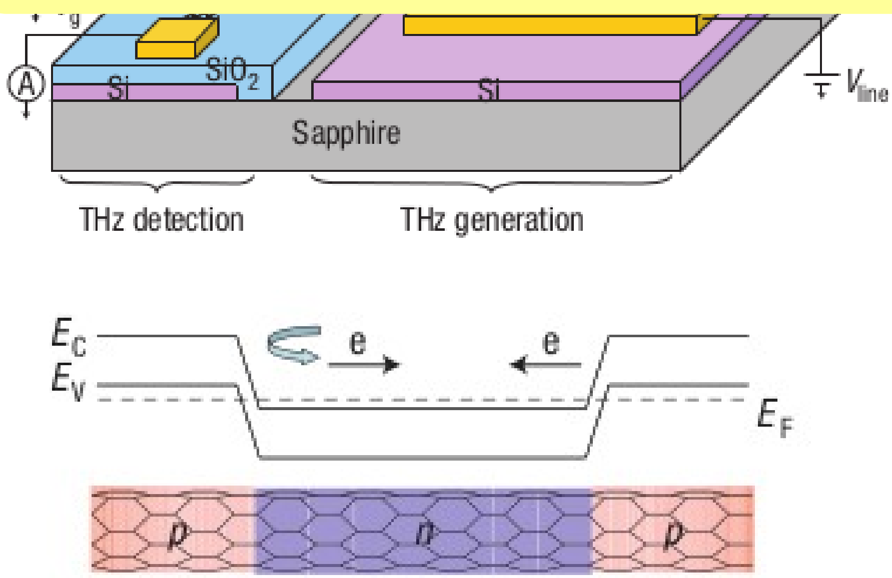


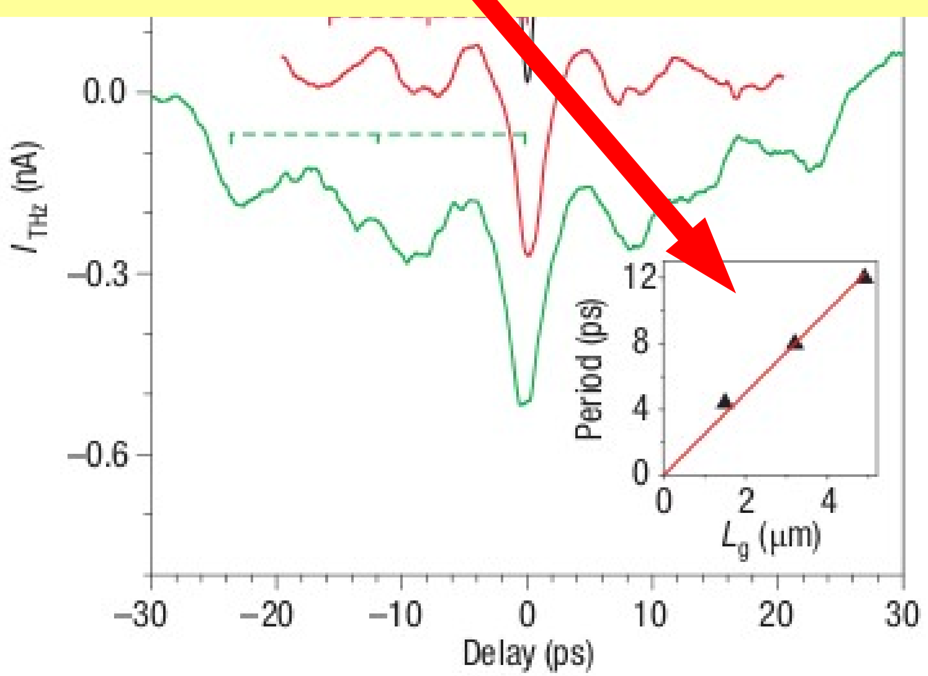
How to observe?

Terahertz time-domain measurement of ballistic electron resonance in a single-walled carbon nanotube

Nature Nano 2008

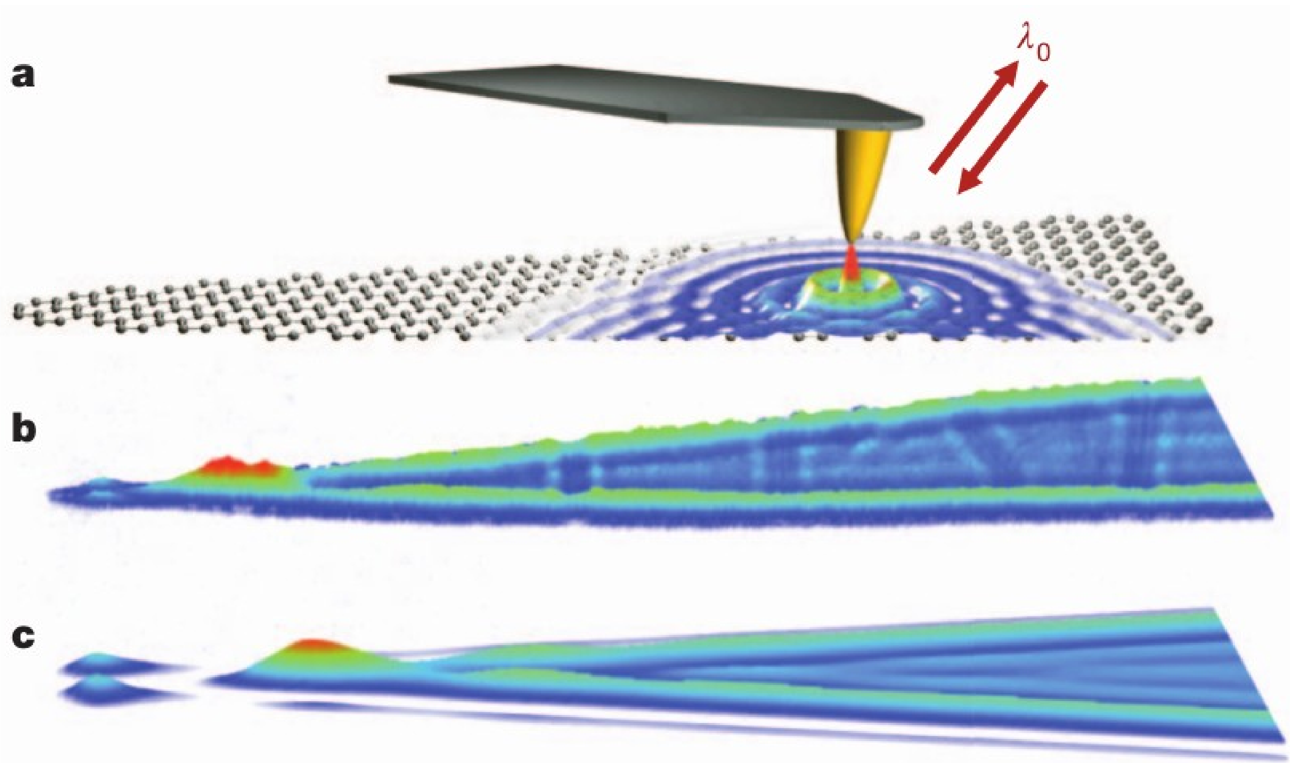
Measured velocity $\approx 10^6 \, m/s$ distinct from plasmon velocity!
Perhaps coincidentally, this equals $v' = \frac{v}{\sqrt{d}}, \quad d=1$

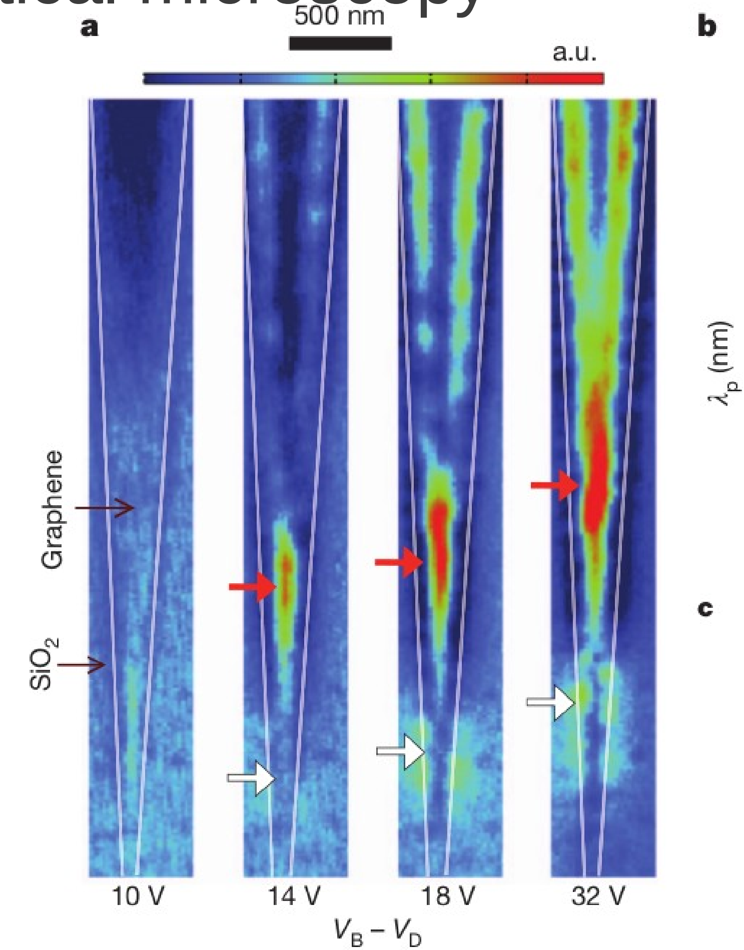




How to observe?

J Chen … F H L Koppens, Nature (2012)
Visualize localized plasmon modes in real space by scattering-type scanning near-field optical microscopy





Summary

1) Wavelike (ballistic) heat propagation at CN

2) Charge-neutral mode, characteristic velocity

$$v' = v/\sqrt{2}$$

3) Exist only at CN (switchable/tunable)

4) frequency range

$$0.2\,THz < f < 3\,THz \qquad 300\,nm < \lambda < 5\,\mu m$$

Textbook: sound is the form of energy that travels in waves