the process characteristics alone does not result in satisfactory control while attempts to control the unknown plant without identification may result in poor response. Hence, according to Feldbaum, the controller A in an automatic control system with incomplete information regarding the plant B must simultaneously solve two problems that are closely related but different in character. Feldbaum referred to this as *dual control.* First, on the basis of the information collected, the controller must determine the characteristics and state of the plant B. Second, on the basis of this acquired knowledge, it has to determine what actions are necessary for successful control. The first problem may be considered one of estimation or identification while the second is one of control.

Two philosophically different approaches exist for the solution of the adaptive control problem discussed earlier. In the first approach, referred to as *indirect control*, the plant parameters are estimated on-line and the control parameters are adjusted based on these estimates. Such a procedure has also been referred to as *explicit identification* in the literature [3]. In contrast to this, in what is referred to as *direct control,* no effort is made to identify the plant parameters but the control parameters are directly adjusted to improve a performance index. This is also referred to as *implicit identification.* In conformity with the ideas expressed by Feldbaum, we note that in both cases efforts have to be made to probe the system to determine its behavior even as control action is being taken based on the most recent information available. The input to the process is therefore used simultaneously for both identification and control purposes. However, not every estimation scheme followed by a suitable control action will result in optimal or even stable behavior of the overall system. Hence, the estimation and control procedures have to be blended carefully to achieve the desired objective. The adaptive control schemes described in the chapters following can be considered special cases where successful dual control has been realized.

In Sections 1.4.1 and 1.4.2, we deal with parameter perturbation and sensitivity methods that are examples of the direct and indirect method respectively. These methods were investigated extensively in the 1960s and represented at that time the two principal approaches to adaptive control. We provide a somewhat more-than-cursory treatment of the two methods in this introductory chapter, since many of the adaptive concepts as well as plant parametrizations suggested later have their origins in these two methods. The reader who is not interested in these historical developments may proceed directly to Section 1.5 with no loss of continuity.

1.4.1 Parameter Perturbation Method

Extremum adaptation mentioned in Section 1.3 was perhaps the most popular among the various adaptive methods investigated in the early 1960s. It had considerable appeal to researchers due to its simplicity, applicability to nonlinear plants, and the fact that it did not require explicit identification of plant parameters. For several years it was investigated extensively and its principal features were studied exhaustively. Starting with the work of Draper and Li [13], who suggested the scheme for optimizing the performance of an internal combustion engine, the method collected a large following, and at present a rather extensive literature exists on this subject. Although its scope is somewhat tangential to that of this book, we nevertheless include some details concerning the method since many of the questions concerning the performance and limitations of adaptive systems encountered at present have counterparts in the analyses of the parameter perturbation method carried out over two decades ago. The interested reader is referred to [12,17,31] for further details.

The parameter perturbation method is a direct-control method and involves perturbation, correlation, and adjustment. Consider, for example, a plant in a laboratory excited by appropriate inputs, having an artificial environment, and provided with a means of continuously measuring a performance index. Further assume that knobs can be twiddled or switches operated to affect this performance index. The question that is posed is how one, asked to adjust the knobs and switches to optimize the performance function, should proceed and what kind of problems the person would face. The most direct and, perhaps, simplest procedure to follow would be to adjust the controls and see the effect on the performance index. If the performance improves, one would continue to alter the controls in the same direction; if it worsens, the controls would have to be changed in the opposite direction. In principle this is what is attempted in the parameter perturbation technique. However, in a practical problem, the number of parameters to be adjusted, the presence of output noise, the fact that parameters of the plant vary with time and that nonlinearities may be present in a plant, and so on, would complicate the problem considerably. Whether or not the method would be considered practically feasible would depend on the stability of the overall system as well as the speed with which it would adapt itself, as the plant parameters varied with time. If the procedure is deemed to be practically feasible, it is certainly simple in concept and easy to implement in terms of hardware.

Detailed analyses of extremal adaptation have been carried out by many authors. The analysis in [23] for example, reveals that sophisticated perturbation methods involving differential equations with multiple time scales would be needed for a precise analysis of the behavior of such systems. However, such approaches are not directly relevant to the contents of this book. We shall, instead, merely bring out the salient qualitative features of the approach as well as its limitations by confining our attention to a few simple problems.

Example 1.1

Consider a no-memory plant in which the input is $\theta(t)$ and the output is a performance function $F(\theta(t))$. $F(\theta)$ is a function of θ as shown in Fig. 1.3. Let θ_{opt} correspond to the value of θ for which $F(\theta)$ has a minimum. Assuming that the designer can only choose the value of θ and observe the corresponding value of $F(\theta)$ at every instant, the objective is to determine a procedure for adjusting $\theta(t)$ so that it converges to the optimal value θ_{opt} of θ.

Let the parameter θ be varied sinusoidally around a nominal value θ_0 so that $\theta(t) = \theta_0 + \epsilon\delta_\theta(t)$, where $\delta_\theta(t) = \sin \omega_p t$. ϵ is the amplitude and ω_p is the frequency of the perturbation. The output $F(\theta)$ will then oscillate around the nominal value $F(\theta_0)$ as $F(\theta_0) + \delta_F(t)$. If $\theta_0 + \epsilon < \theta_{\text{opt}}$, it is obvious that $\theta(t)$ and $F(\theta(t))$ are out of phase while if $\theta_0 - \epsilon > \theta_{\text{opt}}$, they are in phase. Hence, by correlating the perturbation and output as

$$\int_t^{t+2\pi/\omega_p} \delta_\theta(t) F(\theta)\, dt \triangleq R(\theta)$$

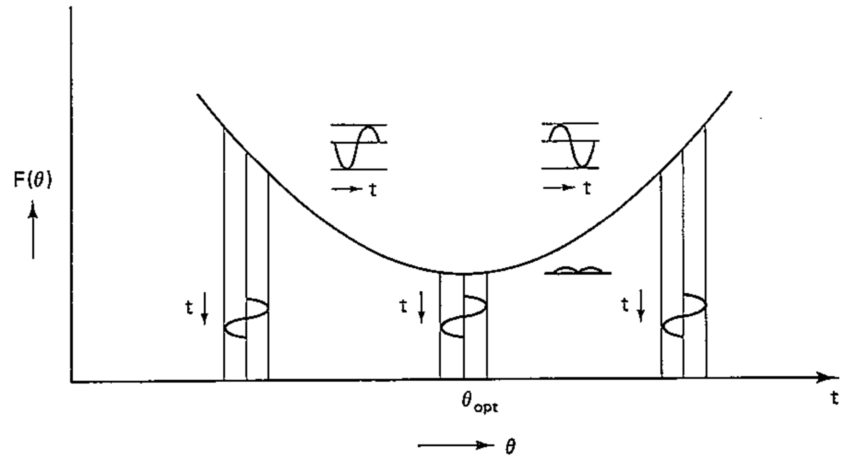

Figure 1.3 Parameter perturbation method – static system.

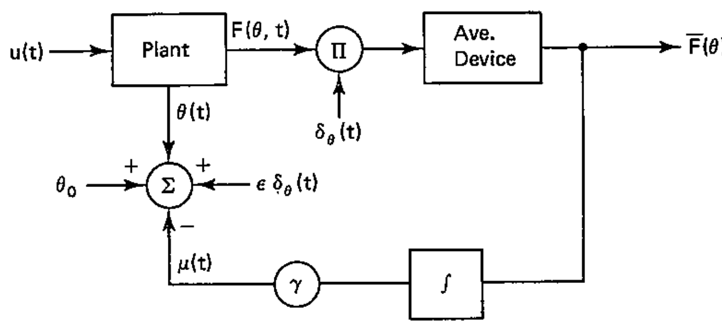

Figure 1.4 Parameter perturbation method – dynamic system.

the side of θ_{opt} on which θ_0 lies can be determined. This in turn indicates whether θ is to be increased or decreased from its nominal value θ_0.

If for every value of θ, the gradient $\nabla_\theta F(\theta)$ with respect to θ is known, it follows directly that if the parameter $\theta(t)$ is adjusted as

$$\dot{\theta}(t) = -\gamma \nabla_\theta F(\theta(t)) \qquad \gamma > 0 \tag{1.8}$$

or alternately

$$\dot{\theta}(t) = -\gamma sgn\left[\nabla_\theta F(\theta(t))\right],$$

where $sgn(x) = +1$ if $x \geq 0$ and -1 otherwise, then $\lim_{t\to\infty} \theta(t) = \theta_{opt}$. Since $\nabla_\theta F(\theta)$ is not known, Eq. (1.8) cannot be implemented in practice. However, the method outlined in Example 1.1 represents an approximation of the above since $R(\theta)$ yields the sign of the gradient and hence can be used for adjusting the parameter.

Example 1.2

Figure 1.4 shows a simple dynamical plant containing a single parameter θ. The input to the plant is u and the only relevant output of the plant is the instantaneous performance index $F(\theta, t)$. It is assumed that using an averaging device, an averaged performance index $\overline{F}(\theta)$ which is independent of time can be obtained. The function $\overline{F}(\theta)$ has the same form as $F(\theta)$ given in Example 1.1. Once again the objective is to determine the optimal value θ_{opt} of θ so that the performance index is minimized.

Note that the problem of optimization of a dynamical system has been reduced to the minimization of the function $\overline{F}(\theta)$ with respect to the parameter θ and that quantities such as the input u are no longer directly relevant. In this equivalent system, the parameter θ can be considered as the input and $\overline{F}(\theta)$ as the output as in Example 1.1.

Assuming once again that $\theta(t)$ is perturbed around a nominal value θ_0 as $\theta(t) = \theta_0 + \epsilon\delta_\theta(t)$, the change in the performance index may be approximated by

$$\overline{F}[t;\theta_0 + \epsilon\delta_\theta(t)] \;\approx\; \overline{F}[t;\theta_0] + \epsilon\,\delta_\theta(t)\nabla_\theta\overline{F}\,|_{\theta=\theta_0}$$

if $\overline{F}$ is a smooth function of the parameter θ. Correlating $\delta_\theta(t)$ and $\overline{F}[t;\theta_0 + \epsilon\delta_\theta(t)]$, we obtain

$$\overline{\delta_\theta(t)\overline{F}[t;\theta(t)]} \;\approx\; \overline{\delta_\theta(t)\overline{F}[t;\theta_0]} + \overline{\epsilon\delta_\theta^2(t)\nabla_\theta\overline{F}\,|_{\theta=\theta_0}} \tag{1.9}$$

where the overbar denotes an average value over an interval of time T. Assuming that $\delta_\theta(t)$ is independent of the input $u(t)$ and has an average value zero, the first term can be neglected. The second term in Eq. (1.9) yields a quantity which is approximately proportional to the gradient of $\overline{F}$ with respect to θ at the operating point θ_0. This quantity is used for updating the parameter θ. At every instant t, the parameter $\theta(t)$ is composed of the nominal value θ_0, the perturbation signal $\epsilon\delta_\theta(t)$, and the correction term $\mu(t)$, which is based on an estimate of the sign of the gradient $\nabla_\theta\overline{F}(\theta)$. $\mu(t)$ is obtained by integrating the output of the averaging device and using a feedback gain γ as shown in Fig. 1.4.

The model above clearly separates the various aspects of the adaptive problem. As described here, the four parameters of interest are the amplitude ϵ of the perturbing signal, its frequency ω_p, the averaging time T, and the gain γ in the feedback path, which can be considered as the step-size of the correction term. A few comments regarding the choice of the values of these parameters are worthwhile [23]:

(i) Too small a value of ϵ makes the determination of the gradient difficult while too large a value may overlook the optimum value.

(ii) A very high frequency of perturbation ω_p may have a negligible effect on the output while a low value of ω_p requires a large averaging time.

(iii) A small value of T may result in a noisy value of the gradient while a large value of T implies slow adaptation.

(iv) A large step size γ may result in hunting or even instability while a small value of γ would result in very slow convergence.

As mentioned previously, the compromises indicated in (i)-(iv) above make their appearances in all adaptive schemes in one form or another and are closely related to the ideas expressed by Feldbaum.

The detailed analysis of even a simple second-order system with a single control parameter reveals that many assumptions have to be made regarding the drift frequency of the control parameter which has to be tracked, the frequency of the correction term in the adaptive parameters, the frequency of the perturbing signal, the bandwidth of the plant and the bandwidth of the closed-loop system. This becomes even more complex if more than one parameter is to be adjusted. Despite these drawbacks, the procedure is intuitively appealing and easy to implement, and consequently finds frequent application in a variety of situations where very little is known about the detailed mathematical model of the plant and only a few parameters can be adjusted. Although such analyses provide valuable insights into the nature of the adaptive process, they are nevertheless beyond the scope of this book.

1.4.2 Sensitivity Method

An alternate approach to the control of systems when uncertainty is present is through the use of sensitivity models. This method gained great popularity in the 1960s and has wide applicability at present in industrial design. A set of forty-five papers, including three surveys containing several hundred references, representative of the state of the art in 1973, was collected in a single volume by Cruz [11], and the reader is referred to it for further information regarding this subject. In this section we briefly indicate how sensitivity methods find application in the design of adaptive control systems.

Since uncertainty in a system can arise in a variety of forms, the corresponding sensitivity questions can also be posed in different ways. Parametric uncertainty may arise due to tolerances within which components are manufactured and the combined effect of variations in parameter values may affect overall system behavior. In adaptive systems where parameters are adjusted iteratively on-line, it would be useful to know how the system performance is improved. The effect of changes in parameters on the eigenvalues of the overall time-invariant system, the states of the system at any instant of time (for example, the terminal time), or the entire trajectory of the system, can be of interest in different design problems. In such cases the partial derivative of the quantity of interest with respect to the parameter that is perturbed has to be computed. Such partial derivatives are called sensitivity functions and assuming they can be determined on-line, the control parameters can be adjusted for optimal behavior using standard hill-climbing methods. Although the use of dynamic models was originally suggested by Byhovskiy [9] in the late 1940s and later extended by Kokotovic [22], Wilkie and Perkins [44], Meissinger [32], and others, in the following examples we present the results obtained independently by Narendra and McBride [34] in the early 1960s and later generalized in [35].

Example 1.3

Consider the linear time-invariant differential equation

$$\ddot{e} + \theta_2 \dot{e} + \theta_1 e = u \qquad e(t_0) = 0 \tag{1.10}$$

Assume that the input u is specified but that the values of the constant parameters θ_1 and θ_2 are to be determined to keep the output $e(t)$ as small as possible. Quantitatively this can be represented as the minimization of a performance function $J(\theta_1, \theta_2)$ with respect to θ_1 and θ_2 where

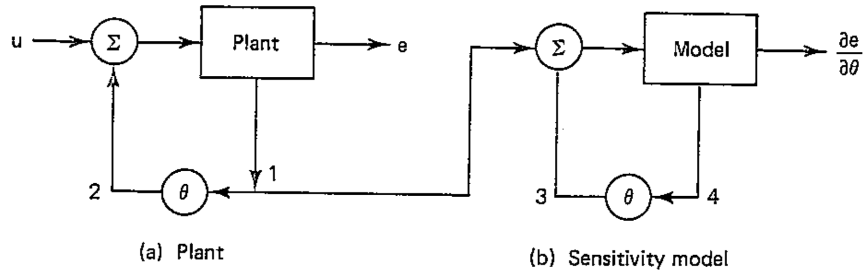

Figure 1.5 Generation of the partial derivative using a sensitivity model.

$$J(\theta_1, \theta_2) = \frac{1}{T}\int_0^T e^2(t)dt.$$

If a gradient approach is to be used to determine the optimal values of θ_1 and θ_2, the partial derivatives $\partial e(t)/\partial\theta_1$ and $\partial e(t)/\partial\theta_2$ are needed. To compute them, we consider the differential equations obtained by taking partial derivatives of the two sides of Eq. (1.10).

$$\begin{aligned}\frac{\partial \ddot{e}(t)}{\partial\theta_1} + \theta_2\frac{\partial \dot{e}(t)}{\partial\theta_1} + \theta_1\frac{\partial e(t)}{\partial\theta_1} &= -e(t)\\ \frac{\partial \ddot{e}(t)}{\partial\theta_2} + \theta_2\frac{\partial \dot{e}(t)}{\partial\theta_2} + \theta_1\frac{\partial e(t)}{\partial\theta_2} &= -\dot{e}(t).\end{aligned}$$

Denoting $\partial e/\partial\theta_1 \triangleq y_1$ and $\partial e/\partial\theta_2 \triangleq y_2$, we have

$$\begin{aligned}\ddot{y}_1 + \theta_2\dot{y}_1 + \theta_1 y_1 &= -e\\ \ddot{y}_2 + \theta_2\dot{y}_2 + \theta_1 y_2 &= -\dot{e}\end{aligned}$$

so that y_1 and y_2 can be generated using models identical to the system in Eq. (1.10) but with inputs $-e$ and $-\dot{e}$ respectively. Such models are referred to as sensitivity models.

Since $\partial e/\partial\theta_1$ and $\partial e/\partial\theta_2$ can be obtained using sensitivity models, the gradient of $J(\theta_1, \theta_2)$ in the parameter space can be obtained as

$$\frac{\partial J}{\partial\theta_i} = \frac{2}{T}\left\{\int_0^T e(t)y_i(t)dt\right\} \qquad i = 1, 2.$$

Hence, by successively adjusting $\theta_i(t)$ over intervals of length T by the algorithms

$$\theta_i(t+T) = \theta_i(t) - \gamma\int_0^T e(t)y_i(t)dt \qquad i = 1, 2$$

or alternately by adjusting it continuously as $\dot{\theta}_i(t) = -\gamma e(t)y_i(t)$, the performance can be improved. When $\theta_i(t)$ converges to some constant value θ_i^*, the optimum parameters are achieved.

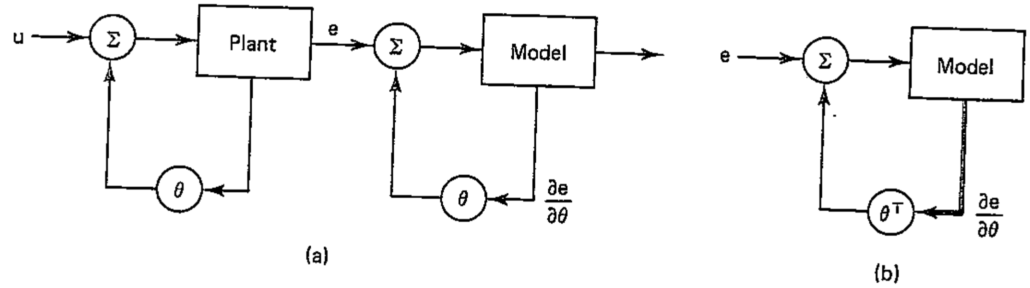

Figure 1.6 Generation of the gradient using a sensitivity model.

Example 1.4 The General Case:

Let a system such as that shown in Fig. 1.5 have u as the input, e an output of interest, and θ an adjustable feedback parameter. It was shown in [35] that if the input signal to the parameter θ in the system denoted by the point 1 is fed into the output of the gain in the sensitivity model whose input (corresponding to u) is identically zero, the output of the model is $\partial e/\partial\theta$. The proof of this is based on the definition of a partial derivative and the linear time-invariant nature of the systems involved. Assuming that the partial derivatives with respect to n parameters $\theta_1, \theta_2, \ldots, \theta_n$ are needed, the same procedure can be repeated using n-sensitivity models of the system. However, it was also shown independently in [22,35] that in special cases where the outputs of all the gains θ_i end in the same summing point, it may be possible to generate all the partial derivatives of y using a single sensitivity model. This is based once again on relatively simple concepts related to linear time-invariant systems.

Since the signal $\partial e/\partial\theta$ can be considered to be obtained as $\partial e/\partial\theta = T_1T_2u$, where T_1 is the transfer function from the point 3 to the output and T_2 is the transfer function from the input of the system to the point 1, and since $T_1T_2 = T_2T_1$, $\partial e/\partial\theta$ also can be generated as the input to the gain parameter θ by making the output e as the input to the model as shown in Fig. 1.6a. This is readily generalized to the many parameter case as shown in Fig. 1.6b since $\partial e/\partial\theta_i, (i = 1, 2, \ldots, n)$ would be the signal generated in the sensitivity model at the input of θ_i. As in the simpler case described earlier, $\partial e/\partial\theta_i$ can be used to generate the gradient of the performance index to adjust the parameters θ_i of the system.

In the discussions thus far, it has been assumed that an exact replica of the overall system can be constructed as a sensitivity model to yield the required sensitivity functions. However, in adaptive systems, which can generally be assumed to be made up of a plant with unknown parameters and a controller whose structure as well as parameters are known, it is no longer a straightforward process to determine the sensitivity model. In such cases, using the input and output of the plant, the parameters of an identification model can be adjusted using the sensitivity approach so that the output of the identification model approximates that of the plant. The sensitivity model for control can then be approximated by the identification model. This makes the sensitivity approach an example of indirect control.

Although simulation results of adaptive systems based on this approach indicate that the convergence of the controller is rapid, very little can be said about the theoretical stability of the overall system. In fact, the problem of assuring stability for arbitrary initial

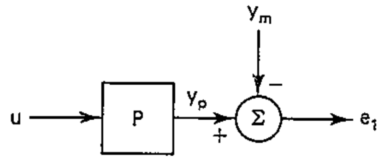

Figure 1.7 The control objective.

conditions encountered in both of the approaches treated in this section led to a search for more systematic procedures for synthesizing stable adaptive controllers and, in turn, resulted in the stability methods discussed in this book.

1.5 MODEL REFERENCE ADAPTIVE SYSTEMS AND SELF-TUNING REGULATORS

As mentioned in Section 1.2, the aim of control is to keep the relevant outputs of a given plant within prescribed limits. If the input and output of a plant P are as shown in Fig. 1.7, the aim of control may be quantitatively stated as the determination of the input u to keep the error $e_1 = y_p - y_m$ between the plant output y_p and a desired output y_m within prescribed values. If y_m is a constant, the problem is one of *regulation* around this value (also known as an operating point or set point). When y_m is a function of time, the problem is referred to as *tracking*. When the characteristics of the plant P are completely known, the former involves the determination of a controller to stabilize the feedback loop around the set point. In the latter case, a suitable controller structure may be employed and control parameters determined so as to minimize a performance index based on the error e_1. As described earlier, powerful analytical techniques based on the optimization of quadratic performance indices of the form of Eq. (1.4) are available when the differential equations describing the behavior of the plant are linear and are known a priori. When the characteristics of the plant are unknown, both regulation and tracking can be viewed as adaptive control problems. Our interest will be in determining suitable controllers for these two cases, when it is known a priori that the plant is linear but contains unknown parameters. *Model reference adaptive systems* (MRAS) and *self-tuning regulators* (STR) are two classes of systems that achieve this objective.

As discussed in Section 1.4, the problem above can be attempted using either an indirect or direct approach. In the indirect approach, the unknown plant parameters are estimated using a model of the plant, before a control input is chosen. In the direct approach, an appropriate controller structure is selected and the parameters of the controller are directly adjusted to reduce some measure of the error e_1. While dealing with the tracking problem, it becomes necessary in both cases to specify the desired output y_m in a suitable form for mathematical tractability. This is generally accomplished by the use of a reference model. Thus, an indirect approach calls for an explicit model of the plant as well as a reference model, while the direct approach requires only the latter. Adaptive systems that make explicit use of such models for identification or control purposes are called MRAS [24]. In view of the important role played by both the identification and reference models in the proper formulation of the adaptive control problem, we provide some additional comments concerning their choice.