Design of Strictly Positive Real Systems Using Constant Output Feedback

C.-H. Huang, P. A. Ioannou, J. Maroulas, and M. G. Safonov

***Abstract*—In this paper, the authors present a linear matrix inequality (LMI) approach to the strictly positive real (SPR) synthesis problem: find an output feedback K such that the closed-loop system $T(s)$ is SPR. The authors establish that if no such constant output feedback K exists, then no dynamic output feedback with a proper transfer matrix exists to make the closed-loop system SPR.**

The existence of K to guarantee the SPR property of the closed-loop system is used to develop an adaptive control scheme that can stabilize any system of arbitrary unknown order and unknown parameters.

***Index Terms*—Adaptive control, H^∞ control, linear matrix inequality, output feedback, positive real functions.**

I. Introduction

The notion of a passive system is one of the oldest in system, circuit, and control theory. Within control theory, a well-known result is that a negative feedback connection of a passive dynamic system and a stable strictly passive uncertainty is internally stable. For finite-dimensional linear time-invariant (LTI) systems, passivity is equivalent to positive realness.

Recently the positive real synthesis problem has been investigated by several researchers (e.g., [1]–[5]). In [6], it has been shown that the strongly positive real synthesis problem is equivalent to a bilinear matrix inequality (BMI) feasibility problem. However, because BMI problems are in general nonconvex and hence difficult to solve [7], [8], there has been much interest in identifying special cases in which the BMI problem can be reduced to a linear matrix inequality (LMI) feasibility problem. So far, this has been possible only in the cases of: 1) full-order control [5] and 2) full-state feedback [3]. A main result of the present paper is the addition of the special case of constant output feedback to the list of positive real synthesis problems that can be solved via LMI's. The result, it turns out, has an interesting application to a problem in adaptive control theory.

We consider the configuration in Fig. 1. This is a special case of that in [1]–[5] in which the original plant matrices $B_1 = B_2$, $C_1 = C_2$, and $D_{ij} = 0$ for $i, j = 1, 2$. We derive an LMI necessary and sufficient condition for the existence of a constant output feedback matrix K for the closed-loop system in Fig. 1 to be strictly positive real (SPR). We also develop a formula for all such K that solves the problem. The derivation leads to a parameterization of all solutions K with only one free matrix which is positive definite. Further, we show that if no constant feedback can lead to an SPR closed-loop system, then no dynamic feedback with proper feedback transfer matrix can do it either. Hence, there exists an output feedback such that the closed-loop system is SPR if and only if there exists a constant output feedback rendering the closed-loop system SPR. Finally, we demonstrate the use of the results by developing an adaptive control scheme that can stabilize and regulate the output of any plant with arbitrary and unknown order and unknown parameters to zero.

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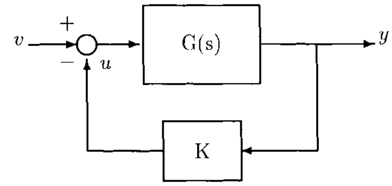

Fig. 1. Closed-loop system $T(s)$.

II. Preliminaries and Notation

Consider the system $T(s)$ shown in Fig. 1. In this figure, K is a constant feedback and $G(s)$ is the transfer function of the system

$$\begin{aligned}\dot{x} &= Ax + Bu\\ y &= C^T x\end{aligned} \tag{1}$$

where $x \in R^n$, $u \in R^q$, $y \in R^q$, and $A \in R^{n\times n}$, $B \in R^{n\times q}$, $C \in R^{n\times q}$ are constant matrices.

The equation of the closed-loop system of Fig. 1 can be expressed as

$$\begin{aligned}\dot{x} &= A_k x + Bv\\ y &= C^T x\end{aligned} \tag{2}$$

where $A_k = A - BKC^T$.

The following definitions and lemmas are referred to in our main result.

Definition 1 [9], [10]: A square transfer function matrix $X(s)$ is SPR if:

1) $X(s)$ is analytic in the closed right-half complex plane;
2) $\mathrm{herm}\{X(j\omega)\} > 0$ for all $\omega \in (-\infty, \infty)$;
3) $\mathrm{herm}\{X(\infty)\} \geq 0$;
4) $\lim_{\omega\to\infty} \omega^2 \mathrm{herm}\{X(j\omega)\} > 0$ if $\mathrm{herm}\{X(\infty)\}$ is singular.

Lemma 1—SPR Lemma [11]–[14]: The closed-loop transfer function matrix $T(s) = C^T(sI - A_k)^{-1}B$ is SPR if and only if there exists a matrix $P = P^T > 0$ such that

$$PA_k + A_k^T P < 0 \tag{3}$$
$$PB = C. \tag{4}$$

Lemma 2—Positive Real Version of the Parrott's Theorem: Let R, U, V, and P be given matrices with appropriate dimensions where U, V^T are full column rank and P is invertible. Then there exists a matrix Q such that

$$\mathrm{herm}\{R + UQV^T\} > 0 \tag{5}$$

if and only if

$$\mathrm{herm}\{U_\perp^T R U_\perp\} > 0, \qquad \mathrm{herm}\{V_\perp^T R V_\perp\} > 0.$$

Moreover, the matrix Q in (5) is given by the equation

$$Q = (I - YV^T(I+R)^{-1}U)^{-1}Y \tag{6}$$

TABLE I
NOTATION

I, I_n	The identity matrix, the $n \times n$ identity matrix.
X^T	Matrix transpose.
X^*	Complex conjugate transpose.
$X^\dagger$	The Moore-Penrose pseudo-inverse of X
$\mathrm{herm}\{\cdot\}$	Hermitian part; $\mathrm{herm}\{X\} \triangleq \frac{1}{2}(X + X^*)$
$X_\perp$	Orthonormal null space of X, $X_\perp^T X = 0$, $[X, X_\perp]$ invertible and $X_\perp^T X_\perp = I$.
$X^{\frac{1}{2}}$	Square root matrix of X such that $X^{\frac{1}{2}T} X^{\frac{1}{2}} = X$
$\stackrel{ss}{=}$	State space realization $G(s) = C(Is - A)^{-1}B + D \stackrel{ss}{=} \begin{bmatrix} A & B \\ C & D \end{bmatrix}$

where

$$\begin{aligned} Y &= -\overline{U}^\dagger (I - L)^{-1} \overline{R} V^{\dagger T} \\ L &= \overline{R V}_\perp \overline{V}_\perp^T \overline{R}^T \overline{U}_\perp \overline{U}_\perp^T \\ \overline{R} &= (I - R)(I + R)^{-1} \\ \overline{U} &= -\sqrt{2}(I + R)^{-1} U \\ \overline{V} &= -\sqrt{2}(I + R^T)^{-1} V. \end{aligned}$$

Proof: See [15]–[17]. ∎

In [18], a formula for all symmetric matrices P satisfying (4) has been introduced. In the following lemma, we develop a formula for all positive definite matrices P satisfying (4).

Lemma 3: Suppose B and C are full rank. Then there exists a matrix $P = P^T > 0$ that satisfies (4) if and only if

$$B^T C = C^T B > 0. \tag{7}$$

Furthermore, when (7) holds, all solutions of (4) are given by

$$P = C(B^T C)^{-1} C^T + B_\perp X B_\perp^T \tag{8}$$

where $X \in R^{n-q \times n-q}$ is an arbitrary positive definite matrix.

Proof: A matrix P satisfies (4) if and only if P can be expressed as

$$P = CB^\dagger + Y B_\perp^T \tag{9}$$

for some $n \times n - q$ matrix Y.

Premultiplying (9) by

$$\begin{bmatrix} B^T \\ B_\perp^T \end{bmatrix}$$

and postmultiplying by its transpose, we obtain

$$\begin{bmatrix} B^T \\ B_\perp^T \end{bmatrix} P [B \quad B_\perp] = \begin{bmatrix} B^T C & B^T Y \\ B_\perp^T C & B_\perp^T Y \end{bmatrix}.$$

Therefore, $P = P^T$ and (4) holds if and only if

$$B^T C = C^T B \tag{10}$$

$$B_\perp^T C = Y^T B \tag{11}$$

and

$$B_\perp^T Y = Y^T B_\perp. \tag{12}$$

Now, the matrix Y satisfies (11) if and only if Y can be expressed as

$$Y = B_\perp Z^T + (B^\dagger)^T C^T B_\perp \tag{13}$$

for some $n - q \times n - q$ matrix Z.

Substituting (13) into (12), we obtain

$$Z = Z^T = B_\perp^T Y. \tag{14}$$

Since we can always choose $Z = Z^T$ so that (10)–(12) are satisfied, we conclude that $P = P^T$ and therefore (4) holds if and only if $B^T C = C^T B$. Now, substituting (13) into (9), all solutions $P = P^T$ of (4) are given by

$$P = CB^\dagger + (B^\dagger)^T C^T B_\perp B_\perp^T + B_\perp Z B_\perp^T \tag{15}$$

where Z is an arbitrary symmetric matrix. Note that (15) has also been introduced in [18].

Further, $P > 0$ if and only if

$$\begin{aligned} \begin{bmatrix} B^T \\ B_\perp^T \end{bmatrix} P [B \quad B_\perp] &= \begin{bmatrix} B^T C & B^T Y \\ B_\perp^T C & B_\perp^T Y \end{bmatrix} \\ &= \begin{bmatrix} B^T C & C^T B_\perp \\ B_\perp^T C & Z \end{bmatrix} > 0. \end{aligned}$$

Applying a Schur complement argument, it can be shown that

$$\begin{bmatrix} B^T C & C^T B_\perp \\ B_\perp^T C & Z \end{bmatrix} > 0$$

if and only if (7) holds and

$$X \triangleq Z - B_\perp^T C (B^T C)^{-1} C^T B_\perp > 0. \tag{16}$$

Since it is always possible to choose Z so that (16) holds, we can conclude that there exists $P = P^T > 0$ such that (4) holds if and only if (7) holds.

Moreover, substituting (16) into (15), all solutions $P = P^T > 0$ to (4) are given by

$$\begin{aligned} P = {} & CB^\dagger + B_\perp B_\perp^T C (B^T C)^{-1} C^T B_\perp B_\perp^T \\ & + (B^\dagger)^T C^T B_\perp B_\perp^T + B_\perp X B_\perp^T \end{aligned} \tag{17}$$

where X is an arbitrary positive definite matrix.

Substituting $B^\dagger = (B^T B)^{-1} B^T$ and $B_\perp B_\perp^T = I - BB^T$ into (17), we obtain (8). ∎

III. All Solutions to the SPR Synthesis Problem

In this section, we develop the necessary and sufficient conditions for the existence of the constant feedback K rendering the closed-loop system with transfer function matrix $T(s)$ in Fig. 1 SPR. Once the Lyapunov matrix P in Lemma 1 is determined, a formula for all solutions K to the SPR synthesis problem is presented. Further, we study the SPR synthesis problem where instead of constant output feedback we use a dynamic one, i.e., the transfer function of the controller is a proper transfer matrix.

Without loss of generality, we assume B and C are full rank.

Theorem 1: There exists a constant matrix K such that the closed-loop transfer function matrix $T(s)$ in Fig. 1 is SPR if and only if

$$B^T C = C^T B > 0 \tag{18}$$

and there exists a positive definite matrix X such that

$$C_\perp^T \mathrm{herm}\{B_\perp X B_\perp^T A\} C_\perp < 0. \tag{19}$$

Furthermore, when (18) and (19) hold, all such solutions K are given by

$$K = C^{\dagger}\mathrm{herm}\{PA\}(I - C_{\perp}(C_{\perp}^T\mathrm{herm}\{PA\}C_{\perp})^{-1} \cdot C_{\perp}^T\mathrm{herm}\{PA\})C^{\dagger T} + S \tag{20}$$

where $P = C(B^TC)^{-1}C^T + B_{\perp}XB_{\perp}^T$ and S is an arbitrary positive definite matrix.

Proof: From Lemma 3, there exists a matrix $P = P^T > 0$ satisfying $PB = C$ if and only if

$$B^TC = C^TB > 0.$$

Further, a formula for P satisfying $PB = C$ is given by

$$P = C(B^TC)^{-1}C^T + B_{\perp}XB_{\perp}^T \tag{21}$$

where X is an arbitrary positive definite matrix.

From Lemma 2, there exists a matrix K such that

$$\mathrm{herm}\{P(A - BKC^T)\} < 0$$

if and only if

$$C_{\perp}^T\mathrm{herm}\{PA\}C_{\perp} < 0 \quad \text{and} \quad (PB)_{\perp}^T\mathrm{herm}\{PA\}(PB)_{\perp} < 0.$$

Since $PB = C$, $\mathrm{herm}\{P(A - BKC^T)\} < 0$ if and only if

$$C_{\perp}^T\mathrm{herm}\{PA\}C_{\perp} < 0. \tag{22}$$

Substituting (21) into (22), we obtain (19).

Now we prove (20). Suppose $B^TC > 0$ and there exists a positive definite matrix X satisfying condition (19), then we can generate P by (21). Further (see (22a), shown at the bottom of the page), $W = (C^TC)^{-(1/2)}C^T\mathrm{herm}\{PA\}C(C^TC)^{-(1/2)} - (C^TC)^{(1/2)}\mathrm{herm}\{K\}(C^TC)^{1/2}$. Applying the Schur complement argument, we can verify that

$$\begin{bmatrix} C_{\perp}^T\mathrm{herm}\{PA\}C_{\perp} & C_{\perp}^T\mathrm{herm}\{PA\}C(C^TC)^{-(1/2)} \\ (C^TC)^{-(1/2)}C^T\mathrm{herm}\{PA\}C_{\perp} & W \end{bmatrix} < 0$$

if and only if

$$C_{\perp}^T\mathrm{herm}\{PA\}C_{\perp} < 0$$

and

$$\Big((C^TC)^{-(1/2)}C^T\mathrm{herm}\{PA\}C(C^TC)^{-(1/2)} - (C^TC)^{1/2}\,\mathrm{herm}\{K\}(C^TC)^{1/2} - (C^TC)^{-(1/2)}C^T \cdot \mathrm{herm}\{PA\}C_{\perp}(C_{\perp}^T\mathrm{herm}\{PA\}C_{\perp})^{-1} \cdot C_{\perp}^T\mathrm{herm}\{PA\}C(C^TC)^{-(1/2)}\Big) < 0. \tag{23}$$

Equation (20) follows. ■

1) Remark: In the single-input/single-output (SISO) case, the necessary condition $B^TC > 0$ implies the relative degree of $G(s)$ is one.

2) Remark: Inequality (19) is essentially an LMI problem which can be solved using the LMI toolbox [19].

Let us now consider the SPR synthesis problem using dynamic output feedback, i.e., we consider

$$u = -H(s)y$$

where

$$H(s) \overset{ss}{=} \begin{bmatrix} A_c & B_c \\ C_c^T & D_c \end{bmatrix}.$$

Theorem 2: If no constant K in Theorem 1 exists, then there exists no dynamic controller with proper transfer matrix which renders the closed-loop system $T(s)$ SPR.

Proof: In the dynamic controller case, the state-space form of the closed-loop system, can be expressed as

$$T(s) \overset{ss}{=} \left[\begin{array}{c|c} \begin{bmatrix} 0 & 0 \\ 0 & A \end{bmatrix} + \begin{bmatrix} I_m & 0 \\ 0 & B \end{bmatrix} Q \begin{bmatrix} I_m & 0 \\ 0 & C^T \end{bmatrix} & \begin{bmatrix} 0 \\ B \end{bmatrix} \\ \hline [0 \quad C^T] & 0 \end{array}\right]$$

where

$$Q = \begin{bmatrix} A_c & B_c \\ C_c^T & D_c \end{bmatrix}$$

and m is the order of the controller.

From Lemma 1, $T(s)$ is SPR if and only if there exists a positive definite matrix P such that

$$P\begin{bmatrix} 0 \\ B \end{bmatrix} = \begin{bmatrix} 0 \\ C \end{bmatrix} \tag{24}$$

and

$$\mathrm{herm}\left\{P\left(\begin{bmatrix} 0 & 0 \\ 0 & A \end{bmatrix} + \begin{bmatrix} I_m & 0 \\ 0 & B \end{bmatrix} Q \begin{bmatrix} I_m & 0 \\ 0 & C^T \end{bmatrix}\right)\right\} < 0. \tag{25}$$

Applying the same technique as in the proof of Theorem 1, we can show that there exists a positive definite matrix P satisfying (24) if and only if

$$B^TC > 0.$$

Moreover, when a solution P exists, then all solutions P to (24) are given as

$$P = \begin{bmatrix} 0 \\ C \end{bmatrix}(B^TC)^{-1}[0 \quad C^T] + \begin{bmatrix} I & 0 \\ 0 & B_{\perp} \end{bmatrix} X \begin{bmatrix} I & 0 \\ 0 & B_{\perp} \end{bmatrix}^T \tag{26}$$

where X is an arbitrary positive definite matrix.

From Lemma 2

$$\mathrm{herm}\left\{P\left(\begin{bmatrix} 0 & 0 \\ 0 & A \end{bmatrix} + \begin{bmatrix} I_m & 0 \\ 0 & B \end{bmatrix} Q \begin{bmatrix} I_m & 0 \\ 0 & C^T \end{bmatrix}\right)\right\} < 0$$

$$\begin{aligned} \mathrm{herm}\{P(A - BKC^T)\} &= \mathrm{herm}\{PA - CKC^T\} \\ &= \left([C_{\perp} \quad C(C^TC)^{-(1/2)}]\begin{bmatrix} C_{\perp}^T \\ (C^TC)^{-(1/2)}C^T \end{bmatrix}\mathrm{herm}\{PA - CKC^T\}[C_{\perp} \quad C(C^TC)^{-(1/2)}] \cdot \begin{bmatrix} C_{\perp}^T \\ (C^TC)^{-(1/2)}C^T \end{bmatrix}\right) \\ &= \left([C_{\perp} \quad C(C^TC)^{-(1/2)}]\begin{bmatrix} C_{\perp}^T\mathrm{herm}\{PA\}C_{\perp} & C_{\perp}^T\mathrm{herm}\{PA\}C(C^TC)^{-(1/2)} \\ (C^TC)^{-(1/2)}C^T\mathrm{herm}\{PA\}C_{\perp} & W \end{bmatrix} \cdot \begin{bmatrix} C_{\perp}^T \\ (C^TC)^{-(1/2)}C^T \end{bmatrix}\right) \end{aligned} \tag{22a}$$

if and only if

$$\begin{bmatrix} I_m & 0 \\ 0 & B \end{bmatrix}_\perp^T \operatorname{herm}\left\{ \begin{bmatrix} 0 & 0 \\ 0 & A \end{bmatrix} P^{-1} \right\} \begin{bmatrix} I_m & 0 \\ 0 & B \end{bmatrix}_\perp < 0 \tag{27}$$

and

$$\begin{bmatrix} I_m & 0 \\ 0 & C \end{bmatrix}_\perp^T \operatorname{herm}\left\{ P \begin{bmatrix} 0 & 0 \\ 0 & A \end{bmatrix} \right\} \begin{bmatrix} I_m & 0 \\ 0 & C \end{bmatrix}_\perp < 0. \tag{28}$$

Also

$$\begin{bmatrix} I_m & 0 \\ 0 & C \end{bmatrix}_\perp$$

can be represented by the following:

$$\begin{bmatrix} I_m & 0 \\ 0 & C \end{bmatrix}_\perp = \begin{bmatrix} 0 \\ C_\perp \end{bmatrix}.$$

Applying (26) to (28), we find that (28) is satisfied if and only if

$$C_\perp^T \operatorname{herm}\{B_\perp X_{22} B_\perp^T A\} C_\perp < 0 \tag{29}$$

where X_{22} is a (2, 2) block of X and is positive definite.

We have shown that $B^T C > 0$ and (29) are the necessary conditions for the existence of the dynamic controller rendering the closed-loop transfer function matrix $T(s)$ SPR. Also $B^T C > 0$ and (29) are the necessary and sufficient conditions given by Theorem 1 for the constant output feedback case. ■

Theorem 2 shows that if we cannot find a constant output feedback controller to make the closed-loop transfer function matrix SPR, then there is no dynamic feedback controller with a proper transfer function matrix that can make the closed-loop transfer function matrix SPR. The results obtained have interesting applications in adaptive control as demonstrated in the following section.

IV. Applications to Adaptive Control Law Design

In this section we apply the results of the previous sections to develop an adaptive control scheme that can stabilize and regulate the output to zero of any plant with arbitrary and unknown order and unknown parameters. The only assumption we use is the existence of a constant output feedback matrix K^* such that the closed-loop transfer function matrix $T(s)$ is SPR. The conditions for existence of K^* are given by Theorem 1. This approach with similar assumptions is not new in adaptive control [20]–[22] and is included here for demonstrating the usefulness of the results obtained.

We can rewrite the closed-loop system (2) as

$$\begin{aligned} \dot{x} &= (A - BK^* C^T)x - B(K - K^*)y \\ y &= C^T x \end{aligned}$$

or

$$\dot{x} = A^* x - B\tilde{K} y, \qquad y = C^T x \tag{30}$$

where $A^* = A - BK^* C^T$, $\tilde{K} = K - K^*$, and $K(t)$ is the estimate of K^* at time t.

We start by considering the quadratic function

$$V = \frac{x^T P x}{2} + \operatorname{trace}\left(\frac{\tilde{K}^T \Gamma^{-1} \tilde{K}}{2} \right)$$

where P satisfies Lemma 1 and Γ is an arbitrary positive definite matrix. The time derivative of V along any trajectory of (30) is given by

$$\dot{V} = x^T (PA^* + A^{*T} P)x - \operatorname{trace}(\tilde{K}^T y y^T - \tilde{K}^T \Gamma^{-1} \dot{\tilde{K}}).$$

If we choose

$$\dot{\tilde{K}} = \Gamma y y^T \tag{31}$$

we have

$$\dot{V} = x^T (PA^* + A^{*T} P)x \leq 0.$$

Since V is a quadratic function and $\dot{V} \leq 0$, we conclude that V is a Lyapunov function for the system (30), (31).

Since V is a nonincreasing function of time, the $\lim_{t \to \infty} V(t)$ exists. Therefore, we obtain x, $\tilde{K} \in L_\infty$ and $x \in L_2$. Since $\dot{K} = \Gamma y y^T$ and $y = C^T x$ where $x \in L_\infty$, we have $\dot{K} \in L_\infty$. Since $\dot{x} \in L_\infty$ due to $\tilde{K}$, x, $y \in L_\infty$, we conclude from $\dot{x} \in L_\infty$ and $x \in L_2$ [23] that $x(t) \to 0$ as $t \to \infty$.

Hence, $u = -K(t)y$ with $\dot{K} = \Gamma y y^T$ can stabilize any system of any order and drive y, x to zero as long as the assumption of the existence of K^* that makes the closed-loop plant transfer matrix SPR is satisfied. Theorem 1 gives necessary and sufficient conditions for this assumption to hold.

V. Conclusion

In this paper we developed necessary and sufficient conditions for the plant state-space matrices that guarantee the existence of a constant output feedback gain matrix K so that the closed-loop system is SPR. The necessary and sufficient conditions are represented in the form of LMI. In addition we developed a procedure for calculating such K from the knowledge of the system matrices. We established that if no such K exists then no dynamic output feedback with proper transfer function matrix can make the closed-loop system SPR.

We showed that the existence of K for the closed-loop system to be SPR can be used to generate an adaptive control regulator that can stabilize any plant with arbitrary order and unknown parameters and regulate its output vector to zero.

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Path-Clearing Policies for Flexible Manufacturing Systems

Kevin Burgess and Kevin M. Passino

***Abstract*— In practical manufacturing settings it is often possible to obtain, in real-time, information about the operation of several machines in a flexible manufacturing system (FMS) that can be quite useful in scheduling part flows. In this brief paper the authors introduce some scheduling policies that can effectively utilize such information (something the policies in [1] do not do) and they provide sufficient conditions for the stability of two such policies.**

***Index Terms*— Boundedness, manufacturing systems, scheduling, stability, traffic control.**

I. Introduction

In this paper, we consider the use of global information for scheduling flexible manufacturing systems (FMS) of the type considered in [1] and later in [2]–[4]. This paper is unique with respect to this body of work in that it provides an analysis of a particular class of "global," rather than "local," scheduling policies. The primary advantage of an FMS composed of individual machines, each with its own scheduling policy that utilizes only local information, is that the individual machines need not communicate with one another so that real-time implementation is simplified. However, for many modern FMS's, it is quite realistic to allow intermachine communications. Here, we seek to exploit this fact by developing scheduling policies that incorporate information from other parts of the network that can be useful in making efficient scheduling decisions. In using more "global" information, we are careful to minimize the level of necessary communications so that our global policies are implementable in real-time, just as the local policies mentioned above.

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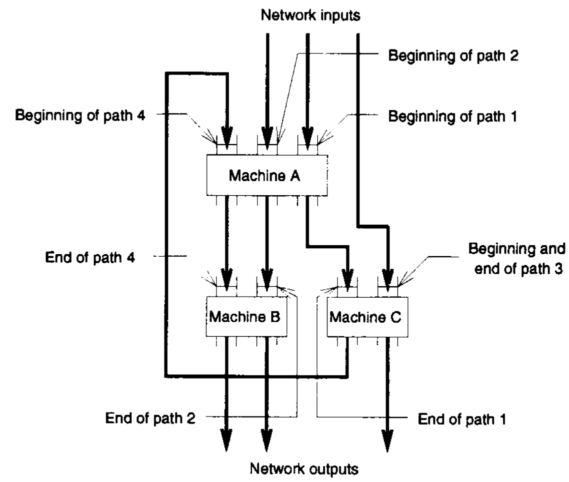

Fig. 1. Example FMS with paths labeled.

In this work, which we view as only a first step toward solving the problem of how to use global FMS information to achieve high-performance scheduling, we define and analyze a class of global scheduling policies that we call path-clearing (PC) policies. Similar to the way in which local policies select a buffer to service from among the buffers of a single machine, PC policies select from among a set of *paths* to service. A path is a set of topologically consecutive buffers which can be serviced simultaneously. In general, a PC policy will choose from among *sets* of paths to process. When a PC policy chooses a set of paths to process, all buffers in each path in the set are processed simultaneously (hence, all paths in a set must be able to be processed at the same time). Once a PC policy begins servicing a set of paths, servicing continues until all paths in the set are clear of parts.

II. System Description and Notation

Let there be N "paths" within the FMS. The three important attributes of any path are: 1) all of its buffers may be serviced at the same time; 2) its buffers are directly connected in the network; and 3) if a buffer is on one path it cannot be on another path, and all buffers must be on a path. For example, in the network of Fig. 1 we can define four paths which begin and end as indicated. Notice that the beginning and end of a path are defined as the first and last buffer in the path, respectively. Let each buffer be referred to by a coordinate (i, j), where i is the path number and j is the buffer