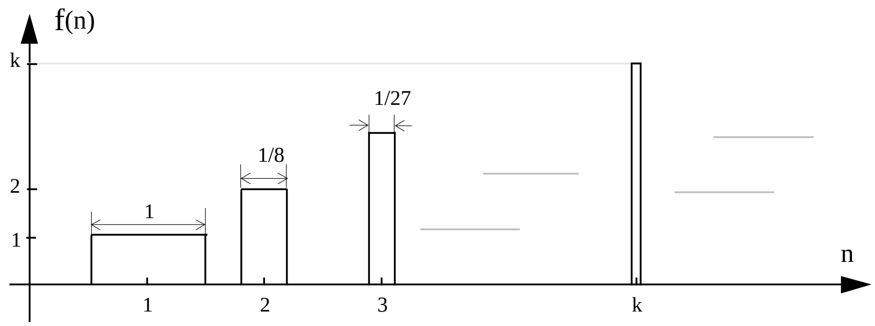
2.S999 ADAPTIVE CONTROL
MIDTERM EXAM
DUE APRIL 10, 11AM

You are free to use class notes and the course text, but no other sources. The work you submit must be your own.

Problem 1 (10 points). Take $u(t) = f(t)$ where $f(t)$, shown in the figure, is a sequence of pulses centered at n with width $\frac{1}{n^3}$ and amplitude n, where $n = 1, 2, ..., \infty$.



(a) Show that $u \in \mathcal{L}_1$ but $u \notin \mathcal{L}_2$ and $u \notin \mathcal{L}_\infty$

(b) If $y = G(s)u$ where $G(s) = \frac{1}{s+1}$ and $u = f$, show that $y \in \mathcal{L}_1 \cap \mathcal{L}_\infty$ and $|y(t)| \to 0$ as $t \to \infty$

Problem 2 (10 points). Prove the following statement: The matrix A is Hurwitz if and only if, given any symmetric positive-definite matrix Q there exists a $P = P^T > 0$ which is the *unique* solution to

$$A^T P + PA = -Q.$$

Problem 3 (30 points). Consider the unknown plant

$$m\ddot{y} + \beta\dot{y} + ky = u$$

where k and β are unknown scalars and m is a known scalar with $m > 0$. The goal is to design u so that y tracks a desired trajectory y_d as $t \to \infty$. Assume that both $\dot{y}_d$ and $\ddot{y}_d$ are available signals. Design an adaptive phase lead controller for the following two scenarios

(a) y and $\dot{y}$ are measurable

(b) Only y is measurable

In both cases, prove that the closed-loop system is stable and show that the tracking goal is achievable.

Problem 4 (30 points). Consider the unknown plant

$$\dot{x} = A_p x + bu + d$$

where $d(t)$ is a bounded time varying disturbance, b is known and A_p is unknown. A_p is an $n \times n$ matrix, b and $d(t)$ are $n \times 1$ vectors, (A_p, b) is controllable. The reference model is chosen as

$$\dot{x}_m = A_m x_m + br$$

where there exists θ^* such that $A_p + b\theta^* = A_m$.

(a) Show that an adaptive controller can be designed using the Projection algorithm that is stable for any bounded disturbance. Prove stability.

(b) Suppose $d(t)$ is such that $bd_0 = d$ where d_0 is a scalar and a constant. Design an adaptive controller that does not require any modification such as a projection, a dead-zone, or a σ-modification. Prove Stability.

(c) Simulate your adaptive controller for the parameters in Table 1 with a time varying reference signal of your choice. Increase the level of the disturbance until your adaptive parameter hits the projection bounds to illustrate the "steel" ball effect of the projection algorithm. You may want to try increasing the amplitude of the reference input to get the adaptive parameters to move faster. One can also add a γ to the update law to increase the learning rate.

Table 1. Simulation Parameters

Paramater	Value
$x(0)^T$	$\begin{bmatrix} 0 & 0.3 \end{bmatrix}$
A_p	$\begin{bmatrix} -1.0 & 0.9 \\ 1 & -0.5 \end{bmatrix}$
A_m	$\begin{bmatrix} -1.0 & 0.9 \\ 0.8 & -1.1 \end{bmatrix}$
b^T	$\begin{bmatrix} 0 & -0.2 \end{bmatrix}$
θ^*	$\begin{bmatrix} 1 & 3 \end{bmatrix}$

Problem 5 (20 points). Consider the unknown scalar system $\dot{x} = ax + b\nu$ where

$$\nu = \text{sat}(u)$$

with

$$\text{sat}(u) = \begin{cases} u_{\max} & \text{if } u \geq u_{\max} \\ u & \text{if } |u| < u_{\max} \\ -u_{\max} & \text{if } u \leq -u_{\max} \end{cases}, \quad \text{and } u_{\max} > 0.$$

Design an adaptive controller so that the plant follows the reference model $\dot{x}_m = a_m x_m + b_m r$ for the following two cases and prove stability. Be sure to emphasize wether it is local or global.

(a) a is unknown but negative and b is known.
(b) a is unknown but positive and b is known.
(c) a and b are unknown and the sign of b is known.